Preface

Here are my online notes for my Algebra course that I teach here at Lamar University, although I have to admit that it's been years since I last taught this course. At this point in my career I mostly teach Calculus and Differential Equations.

Despite the fact that these are my "class notes", they should be accessible to anyone wanting to learn Algebra or needing a refresher for algebra. I've tried to make the notes as self contained as possible and do not reference any book. However, they do assume that you've had some exposure to the basics of algebra at some point prior to this. While there is some review of exponents, factoring and graphing it is assumed that not a lot of review will be needed to remind you how these topics work.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn algebra I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn't covered in class.

2. Because I want these notes to provide some more examples for you to read through, I don't always work the same problems in class as those given in the notes. Likewise, even if I do work some of the problems in here I may work fewer problems in class than are presented here.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible in writing these up, but the reality is that I can't anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I've not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.

Transformations

In this section we are going to see how knowledge of some fairly simple graphs can help us graph some more complicated graphs. Collectively the methods we're going to be looking at in this section are called **transformations**.

Vertical Shifts

The first transformation we'll look at is a vertical shift.

Given the graph of $f(x)$ the graph of $g(x) = f(x) + c$ will be the graph of $f(x)$ shifted up by c units if c is positive and or down by c units if c is negative.

So, if we can graph $f(x)$ getting the graph of $g(x)$ is fairly easy. Let's take a look at a couple of examples.

Example 1 Using transformations sketch the graph of the following functions.

(a) $g(x) = x^2 + 3$ [Solution]

(b) $f(x) = \sqrt{x} - 5$ [Solution]

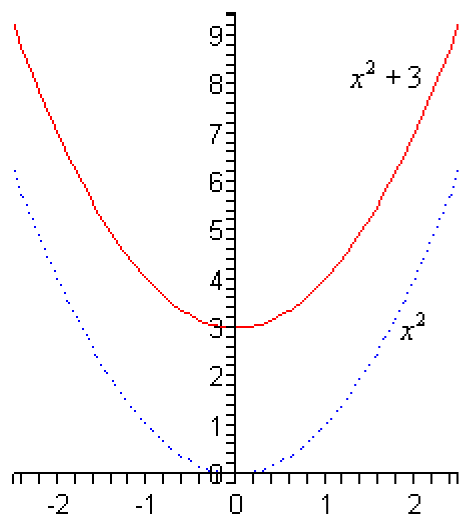
Solution

The first thing to do here is graph the function without the constant which by this point should be fairly simple for you. Then shift accordingly.

(a) $g(x) = x^2 + 3$

In this case we first need to graph x^2 (the dotted line on the graph below) and then pick this up and shift it upwards by 3. Coordinate wise this will mean adding 3 onto all the y coordinates of points on x^2.

Here is the sketch for this one.



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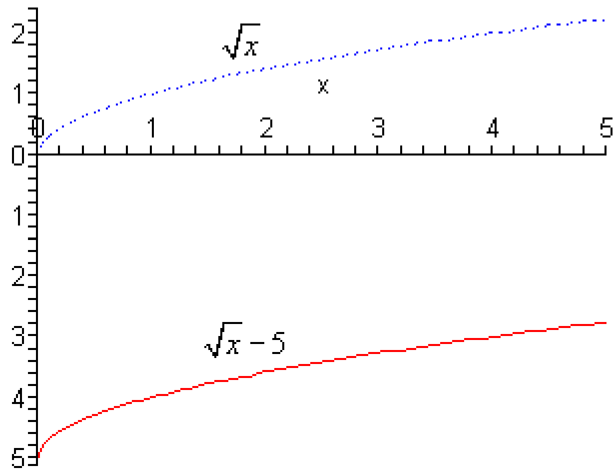
(b) $f(x) = \sqrt{x} - 5$

Okay, in this case we're going to be shifting the graph of $\sqrt{x}$ (the dotted line on the graph below) down by 5. Again, from a coordinate standpoint this means that we subtract 5 from the y coordinates of points on $\sqrt{x}$.

Here is this graph.



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So, vertical shifts aren't all that bad if we can graph the "base" function first. Note as well that if you're not sure that you believe the graphs in the previous set of examples all you need to do is plug a couple values of x into the function and verify that they are in fact the correct graphs.

Horizontal Shifts

These are fairly simple as well although there is one bit where we need to be careful.

Given the graph of $f(x)$ the graph of $g(x) = f(x+c)$ will be the graph of $f(x)$ shifted left by c units if c is positive and or right by c units if c is negative.

Now, we need to be careful here. A positive c shifts a graph in the negative direction and a negative c shifts a graph in the positive direction. They are exactly opposite than vertical shifts and it's easy to flip these around and shift incorrectly if we aren't being careful.

Example 2 Using transformations sketch the graph of the following functions.

(a) $h(x) = (x+2)^3$ [Solution]

(b) $g(x) = \sqrt{x-4}$ [Solution]

Solution

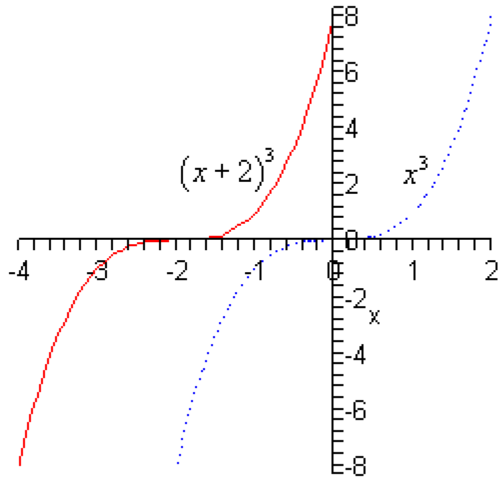
(a) $h(x) = (x+2)^3$

Okay, with these we need to first identify the "base" function. That is the function that's being shifted. In this case it looks like we are shifting $f(x) = x^3$. We can then see that,

$$h(x) = (x+2)^3 = f(x+2)$$

In this case $c = 2$ and so we're going to shift the graph of $f(x) = x^3$ (the dotted line on the graph below) and move it 2 units to the left. This will mean subtracting 2 from the x coordinates of all the points on $f(x) = x^3$.

Here is the graph for this problem.



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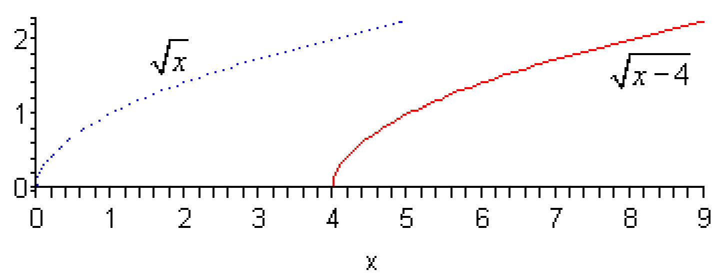
(b) $g(x) = \sqrt{x-4}$

In this case it looks like the base function is $\sqrt{x}$ and it also looks like $c = -4$ and so we will be shifting the graph of $\sqrt{x}$ (the dotted line on the graph below) to the right by 4 units. In terms of coordinates this will mean that we're going to add 4 onto the x coordinate of all the points on $\sqrt{x}$.

Here is the sketch for this function.



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Vertical and Horizontal Shifts

Now we can also combine the two shifts we just got done looking at into a single problem. If we know the graph of $f(x)$ the graph of $g(x) = f(x+c)+k$ will be the graph of $f(x)$ shifted

left or right by c units depending on the sign of c and up or down by k units depending on the sign of k.

Let's take a look at a couple of examples.

Example 3 Use transformation to sketch the graph of each of the following.

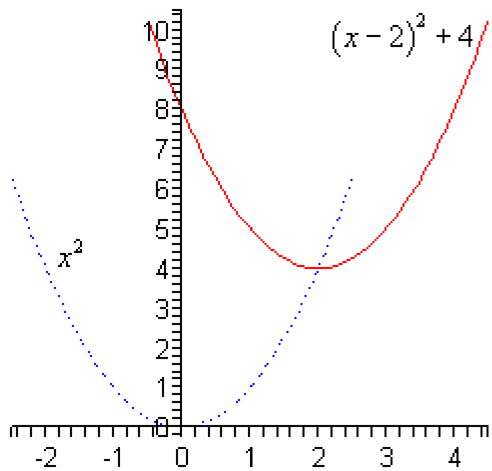
(a) $f(x) = (x-2)^2 + 4$ [Solution]

(b) $g(x) = |x+3| - 5$ [Solution]

Solution

(a) $f(x) = (x-2)^2 + 4$

In this part it looks like the base function is x^2 and it looks like will be shift this to the right by 2 (since $c = -2$) and up by 4 (since $k = 4$). Here is the sketch of this function.



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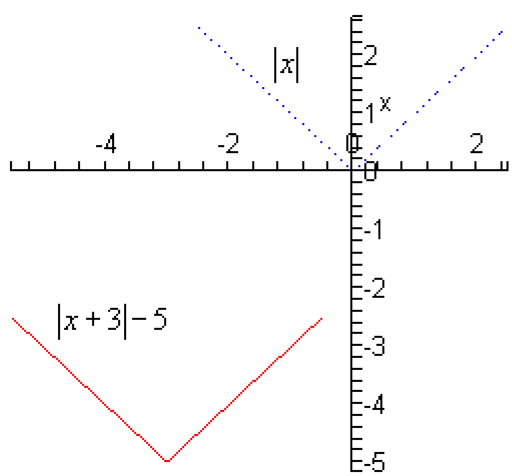
(b) $g(x) = |x+3| - 5$

For this part we will be shifting $|x|$ to the left by 3 (since $c = 3$) and down 5 (since $k = -5$). Here is the sketch of this function.



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Reflections
The final set of transformations that we're going to be looking at in this section aren't shifts, but instead they are called reflections and there are two of them.

Reflection about the *x*-axis.

Given the graph of $f\left(x\right)$ then the graph of $g\left(x\right)=-f\left(x\right)$ is the graph of $f\left(x\right)$ *reflected* about the *x*-axis. This means that the signs on the all the *y* coordinates are changed to the opposite sign.

Reflection about the *y*-axis.

Given the graph of $f\left(x\right)$ then the graph of $g\left(x\right)=f\left(-x\right)$ is the graph of $f\left(x\right)$ *reflected* about the *y*-axis. This means that the signs on the all the *x* coordinates are changed to the opposite sign.

Here is an example of each.

Example 4 Using transformation sketch the graph of each of the following.

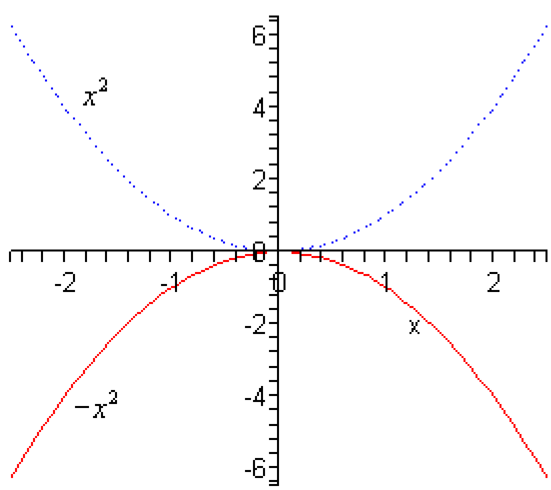
(a) $g\left(x\right)=-x^2$ [Solution]

(b) $h\left(x\right)=\sqrt{-x}$ [Solution]

Solution

(a) Based on the placement of the minus sign (*i.e.* it's outside the square and NOT inside the square, or $\left(-x\right)^2$) it looks like we will be reflecting x^2 about the *x*-axis. So, again, the means that all we do is change the sign on all the *y* coordinates.

Here is the sketch of this graph.



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(b) Now with this one let's first address the minus sign under the square root in more general terms. We know that we can't take the square roots of negative numbers, however the presence of that minus sign doesn't necessarily cause problems. We won't be able to plug positive values of *x* into the function since that would give square roots of negative numbers. However, if *x* were

negative, then the negative of a negative number is positive and that is okay. For instance,

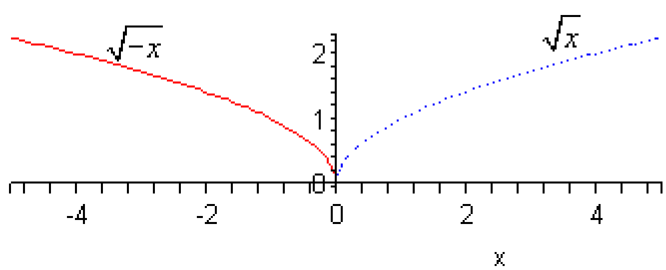
$$h(-4) = \sqrt{-(-4)} = \sqrt{4} = 2$$

So, don't get all worried about that minus sign.

Now, let's address the reflection here. Since the minus sign is under the square root as opposed to in front of it we are doing a reflection about the *y*-axis. This means that we'll need to change all the signs of points on $\sqrt{x}$.

Note as well that this syncs up with our discussion on this minus sign at the start of this part.

Here is the graph for this function.



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