Algebra Mind Maps

By Frank Santos

Algebra Mind Maps *By Frank Santos*

1) *Algebra Fundamentals*
2) *Add/Sub Polynomials*
3) *Scientific Notation*
4) *Exponents*
5) *Mult. Polynomials*
6) *Factoring Polynomials*
7) *Rationals*
8) *Slopes & Linear Eqs.*
9) *Systems of Equations*
10) *Graphing Inequalities*
11) *Relations & Functions*
12) *Radicals*
13) *Solving Quadratic Eqs.*

ALGEBRA MIND MAP NOTES:

√ ***Concepts cover California Algebra 1 State Standards.***
√ ***Mind Maps are non-linear summaries of key ideas of each concept***
√ ***Problems are solved with 2-Column Notes with step by step procedures.***
√ ***Detailed PowerPoint Presentations are available on my school web site.***
√ ***All Mind Maps and PowerPoint Presentations are in draft form and are continually being updated. All comments and suggestions are appreciated and can be sent to me at fsantos@pleasanton.k12.ca.us***

HOW TO USE THIS RESOURCE:

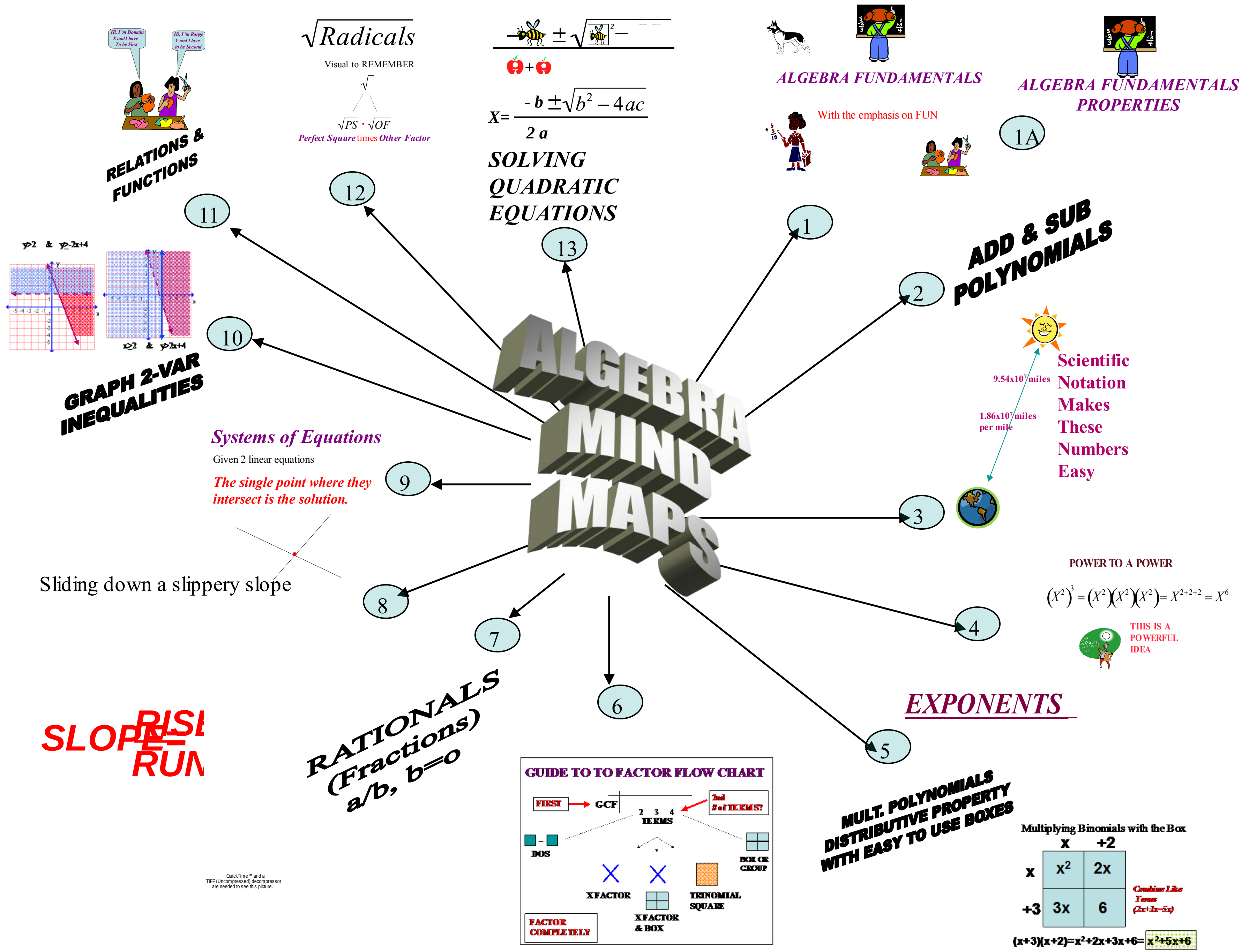
- Find specific concept on appropriate map
- Browse specific maps for overall understanding of general concept
- Get a procedure on specific problems
- Create your own mind map of a concept and compare it with shown map
- Make an overall mind map of algebra with your own knowledge & shown maps

POWERPOINT PRESENTATIONS

Ch1-1 Algebra Fundamentals.pp
Ch1-2,4&5 Algebra Fund 2.ppt
CH1-7 SolveEquations.ppt
CH2-3 AddSub INTEGERS.ppt
CH3-7 AM73&74SolveEqForVar.ppt
CH5-1&2 EXPONENTS.ppt
CH5-4 SciNotation.ppt
CH5-5 DegOfPoly
CH5-5A Terms&Coefficients.ppt
CH5-7 BoxMeth Mult Polys.ppt
CH5-7&8 AddSub Polys.ppt
CH6-1&4 Factoring GCF&TriN.ppt
CH6-1A Factoring Basics.ppt
CH6-2 Factoring Binomial.ppt
CH6-3 Trinomial Square Pres.ppt

CH6-5A Factor ax sq trinomials.ppt
CH6-5B Factor ax sq tri B.ppt
CH6-6 Box Group GCF's.ppt
CH6-7A Factor Strat.ppt
CH6-7B Factor"T"& X".ppt
CH6-7C Factor Strat Summary.ppt
CH6-8 Factoring&Equations.ppt
CH7-1&2 GraphPt&Lines.ppt
CH7-3&5 AM169EqStd.ppt
CH7-3&6 Slope Int Eqs.ppt
CH7-4,5,6&8 Lines&Eq2.ppt
CH7-4,5&6 LinearEq.ppt
CH7-4A Intro Slopes.ppt
CH7-4B SLOPES.ppt
CH7-6A AM170EqGivenSlope&Int.ppt
CH7-6B AM171EqGivenSlope&Pt.ppt
CH7-6C AM172EqGivenTwoPts.ppt
CH7-6D AM174DescribeGraph.ppt
CH7-8APar & Per Lines.ppt
CH7-8C AM173ParPerpLines.ppt

CH8-1 AM181,2&3GraphSys.ppt
CH8-1,2&3 SysLinearEqMM.ppt
CH9-5 GraphLinearIneqMM.ppt
CH10-1,2&3 Rational ExpMM.ppt
CH10-4&5 ADD&SUB RatExp.ppt
CH11-2A RadicalExp2.ppt
CH11-2B RadicalExp2.ppt
Ch11-3,4,5&6RadicalExpMM.ppt
CH11-6&7 Add Radical & PhyT.ppt
CH12-1A RelationsFunctions.ppt
CH12-4 Quad Eq Intro & Graph.ppt
CH12-4A QuadFunctions.ppt
CH12-5&6 DirectIndirectVar.ppt
CH13-1-5 Solve QuadEqMM.ppt
CH13-2 SolveQuad SqRootRule.ppt
Test Prep STATE STAR.ppt
Test Prep District 3rdQ.ppt



ALGEBRA MIND MAPS
ALGEBRA FUNDAMENTALS
With the emphasis on FUN
ALGEBRA FUNDAMENTALS PROPERTIES
1A
1
2
ADD & SUB POLYNOMIALS
Scientific Notation Makes These Numbers Easy
9.54x10^7 miles
1.86x10^7 miles per mile
3
POWER TO A POWER
$(X^2)^3 = (X^2)(X^2)(X^2) = X^{2+2+2} = X^6$
THIS IS A POWERFUL IDEA
4
EXPONENTS
5
MULT. POLYNOMIALS DISTRIBUTIVE PROPERTY WITH EASY TO USE BOXES
Multiplying Binomials with the Box
x
+2
x
x^2
2x
+3
3x
6
Combine Like Terms (2x+3x=5x)
$(x+3)(x+2)=x^2+2x+3x+6=x^2+5x+6$
6
GUIDE TO TO FACTOR FLOW CHART
FIRST
GCF
2nd # of TERMS?
2 3 4 TERMS
DOS
BOX OR GROUP
X FACTOR
X FACTOR & BOX
TRINOMIAL SQUARE
FACTOR COMPLETELY
7
RATIONALS (Fractions) a/b, b≠o
8
SLOPE = RISE / RUN
QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.
Sliding down a slippery slope
9
Systems of Equations
Given 2 linear equations
The single point where they intersect is the solution.
10
GRAPH 2-VAR INEQUALITIES
y>2 & y≥-2x+4
x≥2 & y>2x+4
11
RELATIONS & FUNCTIONS
Hi, I'm Domain X and I have To be First
Hi, I'm Range Y and I love to be Second
12
$\sqrt{Radicals}$
Visual to REMEMBER
$\sqrt{PS} \cdot \sqrt{OF}$
Perfect Square times Other Factor
13
SOLVING QUADRATIC EQUATIONS
$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

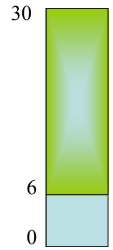
Traditional Method

WHAT PERCENT OF 30 is 6?

$$\frac{Part}{Whole} = \frac{6}{30} = \frac{x}{100}(Percent)$$

$$(100)\frac{6}{30} = \frac{x}{100}(100)$$

$$20\% = x$$



WHAT = X
WHAT % = X/100
OF = • (Times)
IS = = (Equals)

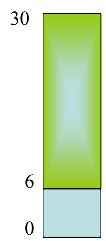
What Percent is What???

WHAT PERCENT OF 30 is 6?

$$\frac{x}{100} \bullet 30 = 6$$

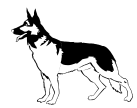
$$\left(\frac{100}{30}\right)\frac{x}{100} \bullet 30 = 6\left(\frac{100}{30}\right)$$

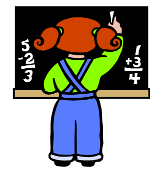
$$x = 20\%$$



Number Sets

$\{1,2,3...\}$	***NATURAL***
$\{0,1,2..\}$	***WHOLE***
$\{...-2,-1,0,1,2,...\}$	***INTEGERS***
$\frac{a}{b}, b \neq 0$	***RATIONAL***
non – repeating, never – ending	***IRRATIONAL***
Rational & Irrational	***REAL***
Negative_SquareRoots	***IMMAGINARY***



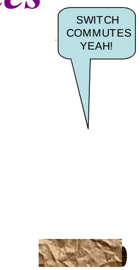


ALGEBRA FUNDAMENTALS

With the emphasis on FUN

Additive & Multiplicative Commutative Properties

(For any rational numbers a & b)



$$a + b = b + a$$

$$a \bullet b = b \bullet a$$

SWITCH PLACES

What about Subtraction & Division?

Adding Fractions

$\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$

- With common denominators
(ADD NUMERATORS)

$\frac{1}{8} + \frac{3}{7} =$

$\frac{7}{56} + \frac{24}{56} = \frac{31}{56}$

- With uncommon denominators
(CONVERT TO COMMON DENOMINATOR AND ADD)

$2\frac{1}{8} + 5\frac{3}{7} =$

$2\frac{7}{56} + 5\frac{24}{56} = 7\frac{31}{56}$

- With mixed numbers
(CONVERT FRACTIONS TO COMMON DENOMINATOR AND ADD WHOLE NUMBERS AND FRACTIONS)

Principle of Balanced Equations

One side of an equal sign is balanced with the other side. Anything that you do to one side of a balanced equation must be done to the other side.

$$x + 3 = 5$$

$$\underline{-3} \quad \underline{-3}$$

$$x = 2$$

Equation Solving & Like Terms

- Maintaining Balance
- Simplify Each Side First
- A Like Term is a number and variable or variables
- Only Like Terms can be combined

$$3x \ \& -5x = -2x$$

$$7y^2 \ \& \ 4y^2 = 11y^2$$

$$56xb^4 \ \& -6xb^4 = 50xb^4$$

Unlike Terms can not be combined

Subtracting Fractions

Same rules but sometimes you have to borrow a whole

$5\frac{1}{7} - 1\frac{3}{7} =$

$5\frac{1}{7} = 4\frac{1}{7} + \frac{7}{7} = 4\frac{8}{7}$

$4\frac{8}{7} - 1\frac{3}{7} = 3\frac{5}{7}$

- BORROWING A WHOLE NUMBER

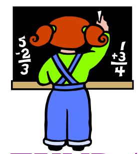
A whole number can be equivalent to a fraction with the same numerator and denominator.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{45}{45} = \frac{1}{1}$$

Using $\frac{7}{7} = 1$ you can subtract the fraction part of the mixed number.

ALGEBRA FUNDAMENTALS PROPERTIES

- *A/M Closure*
- *A/M Commutative*
- *A/M Associative*
- *A/M Identity*
- *Reflexive*
- *Symmetric*
- *Transitive*
- *A/M Inverse*
- *Distributive*



DISTRIBUTIVE Property

(For any rational numbers a & b)

$$a(b+c)=ab+ac$$

Distribute outside term to each term inside the parenthesis

Additive & Multiplicative Closure

(For any rational numbers a & b)

$a \bullet b$ **Is a rational number**

$a+b$ **Is a rational number**

Additive & Multiplicative Inverse Properties

$a+(-a)=0$ **-a is the Additive Inverse**

$a\left(\frac{1}{a}\right)=1$ **1/a is the Multiplicative Inverse**

Think of Inverse as undoing.

QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

Additive & Multiplicative Commutative Properties

(For any rational numbers a & b)

$$a+b=b+a$$

$$a \bullet b=b \bullet a$$

SWITCH PLACES

What about Subtraction & Division?

Properties OF EQUALITY

REFLEXIVE: *a=a is always true*

SYMMETRIC *If a=b then b=a*

TRANSITIVE *If a=b & b=c then a=c*

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Additive & Multiplicative Associative Properties

(For any rational numbers a & b)

$$a+(b+c)=(a+b)+c$$

$$a \bullet (b \bullet c)=a \bullet (b \bullet c)$$

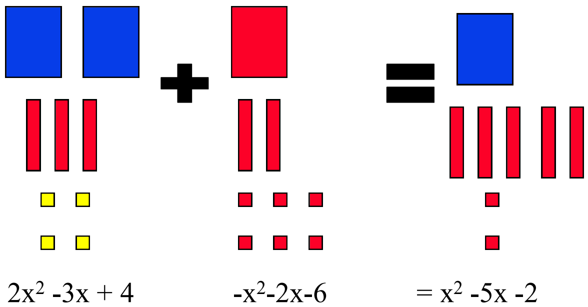
CHANGE ASSOCIATIONS

Additive & Multiplicative Identity Properties

$a \bullet 1=a$ **1 is the Multiplicative Identity**

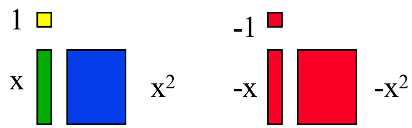
$a+0=a$ **0 is the Additive Identity**

Add Polynomials with Algebra tiles



ALGEBRA TILES

- Makes X's & Algebra Visible
- Combine Like Terms
- Show Factoring



$(\underline{2x^2}\underline{+3x}\underline{-7})+(\underline{-3x^2}\underline{-5x}\underline{+9})$

$+2x^2$	$+3x$	-7
$-3x^2$	$-5x$	$+9$
ANS → $-x^2$	$-2x$	$+2$

ADDING POLYNOMIALS WITH COLUMNS

ADD & SUB POLYNOMIALS

$(2x^2+3x-7) - (\boxed{-3x^2-5x+9})$

$\underline{2x^2} + \underline{3x} - \underline{7} + \underline{3x^2} + \underline{5x} \underline{-9}$

Distribute the -1 (same as the Additive Inverse)

$+2x^2$	$+3x$	-7
$+3x^2$	$+5x$	-9
$5x^2$	$8x$	-16

SUBTRACT POLYNOMIALS WITH COLUMNS

Subtract polynomials

Example: Subtract $(-3x^5 - 5x) - (6x + 8 - 8x^5)$

Since subtraction is the same as "adding the opposite", we can change this problem to addition by changing all of the signs of the polynomial to be subtracted...

$(-3x^5 - 5x) + (-6x - 8 + 8x^5)$

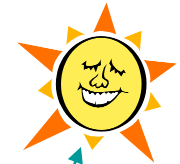
Now, find your like terms…

$-3x^5 - 5x + -6x - 8 + 8x^5$

$5x^5 - 11x - 8$

ANOTHER METHOD

Scientific Notation Makes These Numbers Easy



9.54x10^7 miles

1.86x10^7 miles per second

Changing from Standard Notation to Scientific Notation

Ex. 6800

6800 (3 2 1)

68×10^3

1. Move decimal to get a single digit # and count places moved
2. Answer is a single digit number times the power of ten of places moved.

If the decimal is moved left the power is positive.

If the decimal is moved right the power is negative.

Changing from Scientific Notation to Standard Notation

Ex. 4.5×10^{-3}

00045 (3 2 1)

1. Move decimal the same number of places as the exponent of 10.
(Right if Pos. Left if Neg.)

What is Scientific Notation

A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (eg, 7000 = 7 x 10^3 or 0.0000019 = 1.9 x 10^{-6})

Why do we use it?

It's a shorthand way of writing very large or very small numbers used in science and math and anywhere we have to work with very large or very small numbers.

Multiply two numbers in Scientific Notation

$(3 \times 10^4)(7 \times 10^{-5})$

$= (3 \times 7)(10^4 \times 10^{-5})$

$= 21 \times 10^{-1}$

$= 2.1 \times 10^0$

or 2.1

1. Put #'s in ( )'s Put base 10's in ( )'s
2. Multiply numbers
3. Add exponents of 10.
4. Move decimal to put Answer in Scientific Notation

DIVIDE USING SCIENTIFIC NOTATION

$$\frac{6.20 \times 10^{-5}}{8.0 \times 10^3}$$

$$\frac{6.20}{8.0} \quad \frac{10^{-5}}{10^3}$$

$= 0.775 \times 10^{-8}$

$= 7.75 \times 10^{-9}$

1. *Divide the #'s & Divide the powers of ten (subtract the exponents)*
2. *Put Answer in Scientific Notation*

Addition and subtraction Scientific Notation

$2.0 \times 10^2 + 3.0 \times 10^3$

$.2 \times 10^3 + 3.0 \times 10^3$

$= .2+3 \times 10^3$

$= 3.2 \times 10^3$

1. Make exponents of 10 the same
2. Add 0.2 + 3 and keep the 10^3 intact

The key to adding or subtracting numbers in Scientific Notation is to make sure the exponents are the same.

$2.0 \times 10^7 - 6.3 \times 10^5$

$2.0 \times 10^7 - .063 \times 10^7$

$= 2.0 - .063 \times 10^7$

$= 1.937 \times 10^7$

1. Make exponents of 10 the same
2. Subtract 2.0 - .063 and keep the 10^7 intact

Ex. 6800

Changing from Standard Notation to Scientific Notation

6800.

3 2 1

1. Move decimal to get a single digit # and count places moved

6.8×10^{3}

2. Answer is a single digit number times the power of ten of places moved.

If the decimal is moved left the power is positive.

If the decimal is moved right the power is negative.

What is Scientific Notation

A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (eg, $7000 = 7 \times 10^{3}$ or $0.0000019 = 1.9 \times 10^{-6}$)

Why do we use it?

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Compare these two cases

–6 IS THE BASE

$$(-6)^2 = -6 \bullet -6 = +36$$

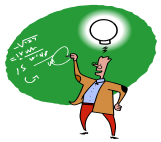
$$-6^2 = -(6)^2 = -(6) \bullet (6) = -36$$

6 IS THE BASE (THE NEG. SIGN IS NOT PART OF THE BASE, IT MUST REMAIN PART OF THE ANSWER)

Anything to the zero power equals 1

POWER TO A POWER

$$(X^2)^3 = (X^2)(X^2)(X^2) = X^{2+2+2} = X^6$$



THIS IS A POWERFUL IDEA

EXPONENTS

2 IS THE EXPONENT

$$X^2$$

X IS THE BASE

INVERTING A NEGATIVE EXPONENTS CHANGES ITS SIGN

$$X^{-2} = \frac{1}{X^2} \qquad \frac{1}{X^{-2}} = X^2$$

$$\left(\frac{3x^3y}{7z^2}\right)^4$$

Take A Power To A Power

$$\left(\frac{3x^3y}{2z^2}\right)^4 = \frac{3^4x^{3\bullet4}y^{1\bullet4}}{2^4z^{2\bullet4}}$$

$$= \frac{3^4x^{12}y^4}{2^4z^8} = \frac{81x^{12}y^4}{16z^8}$$

1. Multiply the outside power to the inside powers.
2. Simplify

MULTIPLYING EXPONENTS

ADD THE EXPONENTS

$$X^A \bullet X^B = X^{A+B}$$

DIVIDING EXPONENTS

SUBTRACT THE EXPONENT OF THE DENOMINATOR FROM THE EXPONENT OF THE NUMERATOR

$$\frac{X^A}{X^B} = X^{A-B}$$

Introducing the Multiply Box

a & b are factors

	b
a	ab

a·b is the product

$a_x b = ab$

	$6x^{-3}$
$7x^8$	$42x^5$

$7x^8$¥$6x^{-3}=42x^5$

Multiply a Binomial by a Trinomial

	c	+d	+e
a	ac	ad	ae
b	bc	bd	bc

(a+b)(c+d+e)= ac+ad+ae
bc+bd+be

Multiplying Binomials with the Box

	x	+2
x	x^2	2x
+3	3x	6

Combine Like Terms (2x+3x=5x)

$(x+3)(x+2)=x^2+2x+3x+6=$ $\boxed{x^2+5x+6}$

MULT. POLYNOMIALS DISTRIBUTIVE PROPERTY WITH EASY TO USE BOXES

HOW TO USE BOX METHOND

1. *Draw a box matrix to match # of polynomial terms*
2. *Put the 1st polynomial down the left side*
3. *Put the 2nd polynomial on top*
4. *Multiply #'s and add exponents of similar bases*
5. *Combine Like Terms*
6. *Write Answer in descending order*

DISTRIBUTIVE PROPERTY

	x	+5
2	2x	10

2(x+5)=2x+10

Factor $16x^4-36$

$\boxed{16x^4} - \boxed{36}$

$a=4x^2$ $b=6$

$$=(4x^2+6)(4x^2-6)$$

Factoring a Difference of Squares

1. Recognize Both Terms are Squares
(a is the sq. root of the first square)
(b is the sq. root of the second square)

2. Use the Diff. Of Squares Formula

$$a^2-b^2=\boxed{(a+b)(a-b)}$$

GUIDE TO TO FACTOR FLOW CHART

FIRST → GCF

2nd # of TERMS?

2 3 4
TERMS

DOS

BOX OR GROUP

X FACTOR

X FACTOR & BOX

TRINOMIAL SQUARE

FACTOR COMPLETELY

Factor x^2+5x+6	Factor x^2+bx+c
6 (top), 2 (left), 3 (right), 5 (bottom); $(x+2)(x+3)$	*1. Put c on top of "X" factor (c is the last #)* *2. Put b on bottom (b is the middle #)* *3. Determine side factors* *4. Put side factors in Answer* Note: Answer will be in the form of (x)(x)

The Factor "X" (Used in Factoring)

a, b — *Given a & b on the sides of the x factor*

ab — *a•b is the top of the x factor*

a, b

a+b — *a+b is the bottom of the x factor*

FACTORING: (OPPOSITE OF MULT)

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

X-BOX

Factor $36x^2+60x+25$	Factoring ax^2+bx+c Trinomials
900 (top), 30 (left), 30 (right), 60 (bottom)	*1. Put ac on top of "X" factor* *2. Put b on bottom of "X" factor* *3. Determine what factors multiply to ac and sum to b.* *4. Put ax^2 in the first box & c in last box* *5. Write factors in other boxes with X's* *6. Determine GCF of upper boxes* *7. Answer is Top Factors times Side Factors.*

	6x	+5
6x	$36x^2$	30x
5	30x	25

$$=(6x+5)(6x+5)=(6x+5)^2$$

How to recognize a trinomial square:

The Middle Term
Can be ±

$$\boxed{9x^2}\pm12x+\boxed{4}$$

a=3x b=2

The Middle Term

$$2•a•b = 2(3x)(2) = 12x$$

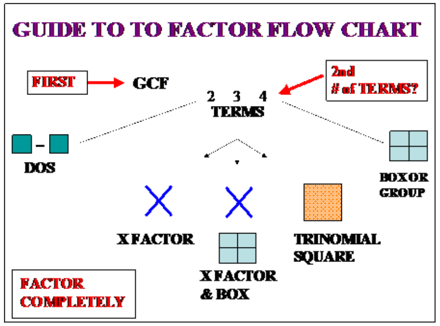
2. The Middle Term is A POSITIVE TWO TIMES a •b OR NEGATIVE TWO TIMES a •b.

Factor $9x^2+12x+4$	Factor trinomial square
$\boxed{9y^2}-12y+\boxed{9}$ a=3y b=3 2•a•b = 2(3y)(3)=12y	1. Recognize it is a trinomial square • First term is a square • Last term is a square • Middle term is 2•A•B 2. Use the Trinomial Square formula $a^2+2ab+b^2=(a+b)^2$ OR $a^2-2ab+b^2=(a-b)^2$ *3. Check answer by multiplying.*

	3y	-3
3y	$9y^2$	-6y
-3	-6y	4

$$=(3y-3)^2$$

Factor $64x^5-4x$	Factor Completely
$64x^5-4x$ $4x\left[\boxed{16x^4}-\boxed{1}\right]$ $a=4x^2$ b=1 $=4x(4x^2+1)(\boxed{4x^2}-\boxed{1})$ a=2x b=1 $=4x(4x^2+1)(2x+1)(2x-1)$	1. Check for GCF • Check for further factoring 2. Use DOS Formula $a^2-b^2=(a+b)(a-b)$ • Check for further factoring 3. Use DOS Formula again $a^2-b^2=(a+b)(a-b)$ • Check for further factoring 4.. *LAST STEP IS* • Check for further factoring



FACTORING:
(OPPOSITE OF MULT)

Simplify $\dfrac{y^2+3y+2}{y^2-1}$

Simplifying Polynomials

$$\frac{y^2+3y+2}{y^2-1}=\frac{(y+2)(y+1)}{(y+1)(y-1)}$$

1. Factor numerator & denom.

2 1 2 3

$$\frac{(y+2)\cancel{(y+1)}}{\cancel{(y+1)}(y-1)}=\frac{(y+2)}{(y-1)}$$

2. Reduce

For y=1 this algebraic expression is UNDEFINED

3. ***Name undefined values***
   (Where denominator is 0)

Multiply Rationals

$$\frac{(3x^2+x)}{14}\bullet\frac{2}{x}$$

$$\frac{x(3x+1)}{14}\bullet\frac{2}{x}$$

$$\frac{\cancel{x}(3x+1)}{{}_7\cancel{14}}\bullet\frac{\cancel{2}^{\,1}}{\cancel{x}}$$

$$\frac{(3x+1)}{7}$$

1. Factor each numerator and denominator if possible.
2. Cancel and/or Reduce

Divide Rationals

$$\frac{x+1}{x^2-1}\div\frac{x+1}{x^2-2x+1}$$

$$\frac{x+1}{x^2-1}\bullet\frac{x^2-2x+1}{x+1}$$

$$\frac{x+1}{(x+1)(x-1)}\bullet\frac{(x-1)(x-1)}{x+1}$$

$$\frac{\cancel{x+1}}{\cancel{(x+1)}\cancel{(x-1)}}\bullet\frac{\cancel{(x-1)}(x-1)}{x+1}$$

$$=\frac{x-1}{x+1}$$

1. INVERT 2nd Fraction & MULTIPLY
2. FACTOR each numerator and denominator if possible.
3. SIMPLIFY

Factor & Reduce

RATIONALS (Fractions) a/b, b≠0

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Adding Rational Expressions with common denominators

$$\frac{p+7}{p-3}-\frac{-p^2+5p+4}{p-3}$$

$$=\frac{p+7}{p-3}-\frac{(-p^2+5p+4)}{p-3}$$

$$=\frac{p+7+p^2-5p-4}{p-3}=\frac{p^2-4p+3}{p-3}$$

$$=\frac{\cancel{(p-3)}(p-1)}{\cancel{p-3}}=\boxed{p-1}$$

3 -3 -1 -4

1. ***ADD Numerators with ( )'s***
   (Distribute the -1)
   Keep the common denominator.
2. ***FACTOR*** & ***SIMPLIFY***
   (If possible)

Adding Rationals with uncommon denominators

$$\frac{2}{x}+5$$

$$=\frac{2}{x}+\frac{x}{x}\cdot 5$$

$$=\frac{2}{x}+\frac{5x}{x}=\frac{5x+2}{x}$$

The Key is to:

1. Make Denominators Common

2. Add Numerators & Simplify

Solving Rational Equations (2nd Deg)

$$x+\frac{6}{x}=-5$$

$$x\left(x+\frac{6}{x}\right)=(-5)x$$

$$x^2+6=-5x$$

$$x^2+5x+6=0$$

6 3 2 5

$$(x+3)(x+2)=0$$

$$x+3=0 \qquad x+2=0$$

$$x=-3 \qquad x=-2$$

1. Bust fractions by multiplying bymon denominator.
2. Set Eq =0
3. Factor
4. Set Each Factor = 0 & Solve

Find the slope of a line Through (3,4) &(-2,6)

$x_1\ y_1 \qquad x_2\ y_2$

(3,4) & (-2,6)

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{-2-3}$$

$$\boxed{\frac{6-4}{-2-3} = \frac{+2}{-5} = -\frac{2}{5}}$$

Find the Slope of a line between 2 Points

1. ***Write x_1, y_1, x_2, y_2 over numbers***
2. ***Write Formula and Substitute x_1, x_2, y_1, y_2 values.***
3. ***Calculate & Simplify***

Write this equation in Standard Form Ax+By=C

Ex #4. $y = \frac{3}{8}x + 4$

$$8\left[y = \frac{3}{8}x + 4\right]8$$

8y=+3x+32

-3x -3x

$\boxed{-3x+8y=32}$ *or* $\boxed{+3x-8y=-32}$

Obj. 169

Write Equation in Standard Form or Slope Intercept Form

1. Bust Fractions by multiplying by LCD
2. ***Move x-term to LHS (Left Hand Side) of equation***

Note: Mult. both sides of the Eq. By (-1) to get an equivalent ans.

Comparison of AM 170, 171 & 172

SUMMARY AM 170

Write Equation of a Line Given Slope and y-intercept

1. ***PUT M&B INTO Y=MX+B FOR EQUATION***

y=mx+b

$\boxed{y=4x+8}$

SUMMARY AM 171

Write Equation of a Line Given Slope and a point

1. ***USE Y=MX+B & SOLVE FOR B***
2. ***PUT M&B INTO Y=MX+B FOR EQUATION***

x y

(4,3)

y=mx+b

Slope of 2

1. 3=2(4)+b
3=8+b
-8 -8
-5=b

2. $\boxed{y=2x-5}$

SUMMARY AM 172

Write equation of a line given two points on the line.

1. ***Put points into to find slope.*** $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. ***USE Y=MX+B & SOLVE FOR B***
3. ***PUT M&B INTO Y=MX+B FOR EQUATION***

(4,3) (6,7)

1. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{6-4} = \frac{4}{2} = 2$

x y

(4,3)

y=mx+b

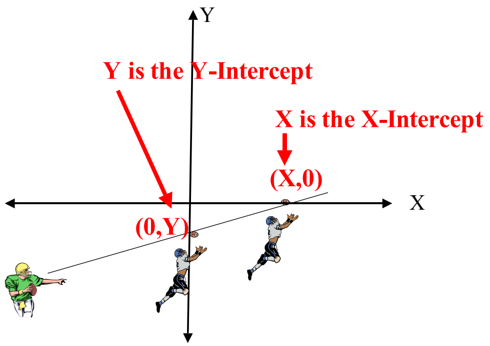
Slope of 2

2. 3=2(4)+b
3=8+b
-8 -8
-5=b

3. $\boxed{y=2x-5}$

Sliding down a slippery slope

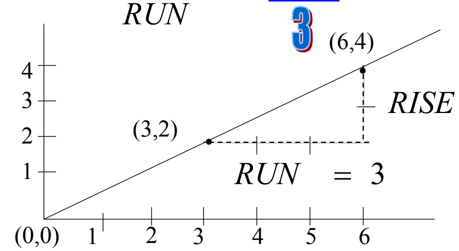
Y & X-Intercepts



SLOPE is a measure of STEEPNESS

$SLOPE = \frac{RISE}{RUN}$

$$SLOPE = \frac{RISE}{RUN} = \frac{2}{3}$$



Find the eq. of a line Perpendicular to y=-2x+6 and Through Point (5,1)

x y

y=-2x+6

Slope Is -2
Neg Recip
Is $^1/_2$

$1 = \frac{1}{2}(5) + b$

$1 = \frac{5}{2} + b$

-5/2 -5/2

$\frac{-3}{2} = b$

$y = \frac{1}{2}x + \frac{-3}{2}$

Equations of Perpendicular Lines

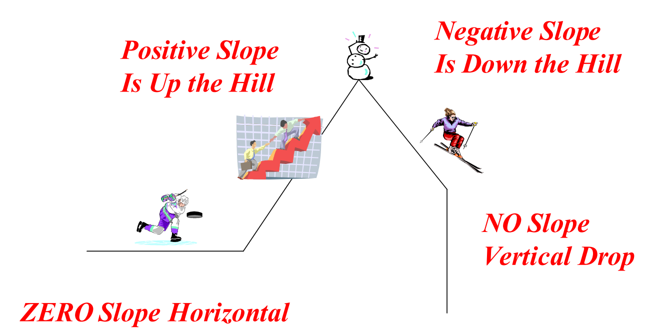
1. Solve the equation for y to find slope. *(Use Neg Recip)*
2. Put slope & (x,y) point into y=mx+b to find b.
3. ***Put m&b (from steps 1&2) into y=mx+b***

Equations of a Line

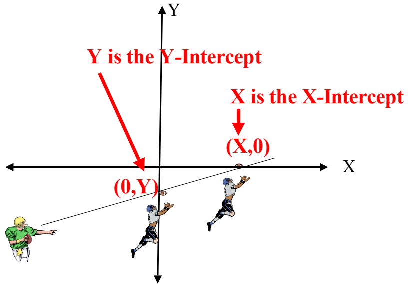
There are ***3 Forms*** of Line Equations

- Standard Form: ***ax+by=c***
- Slope Intercept Form: ***y=mx+b***
- Point-Slope Form ***$y-y_1=m(x-x_1)$***

All 3 describe the line completely but are used for different purposes. You can convert from one form to another.

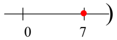


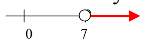
Y & X-Intercepts



Review of an Equation & It's Solution

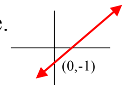
SOLUTIONS ARE THE GRAPH

The graph of one variable equation is a number on the number line. (3x=21 ) x=7

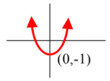
The graph of an inequality is a dot and heavy line & arrow on a number line. (3x>21 ) x>7

The graph of a linear equation is a line.

y=x-1



The graph of a quadratic equation is a parabola. $y=x^2-1$



$$y=-x+3$$
$$y=x-1$$

AM 186&7 SOLVING Systems of Eq. By ELIMINATION/ADD

$$y=-x+3$$
$$1=-2+3$$
$$1=1$$
$$y=x-1$$
$$1=2-1$$
$$1=1$$

$\boxed{(2,1)}$

1. Line up equation variables and #.
2. Combine Like Terms
3. Solve for 1 variable.
4. Put answer into either equation and solve for the other variable.
5. ***CHECK ANS. BY PUTTING ANS. BACK INTO EACH EQ.***

Is (5,4) a solution?

$$y=-x+3$$
$$y=x-1$$

Determine if a given point is a Solution to a Sys of Eq.

$4=-5+3?$ ***Not True***

$4=5-1?$ ***True***

(5,4) IS NOT A SOLUTION

1. Put (x,y) point into each equation.
2. If both equations are true the point is a solution.

Is (2,-3) a solution of

y = 2x-7

SOLUTIONS OF a LINEAR EQUATION

y = 2x-7

-3=2(2)-7

-3=4-7

-3=-3

SO (2,-3) IS A SOLUTION

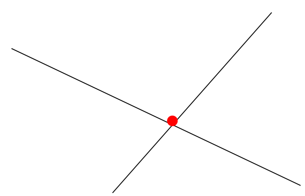
1. Put (2,-3) (x,y) values into the equation.
2. *IF THE EQUATION IS TRUE THE POINT (2,-3) IS A SOLUTION.*

If the equation is not true the point isn't a solution

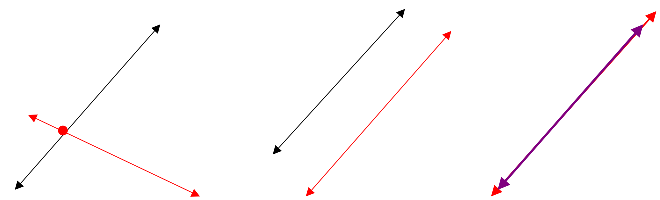
Systems of Equations

Given 2 linear equations

The single point where they intersect is the solution.



Systems of Equations Have 3 Possible Answers



ONE	NONE	INFINITE
(Lines Intersect)	(Lines are Parallel)	(2 lines on each other)

Solve: Find (x,y) Point where these Two lines cross.

$$4y=-2x+2$$
$$y=x+5$$

AM 186&7 SOLVE Systems of Eq. By ELIM./ADD

$$4y=-2x+2$$
$$y=x+5 \leftarrow \text{x2}$$

$$4y=-2x+2$$
$$\underline{2y=2x+10}$$
$$6y=12$$
$$y=2$$
$$2=x+5$$
$$\underline{-5} \quad \underline{-5}$$
$$-3=x$$

(-3,2) ← ANSWER

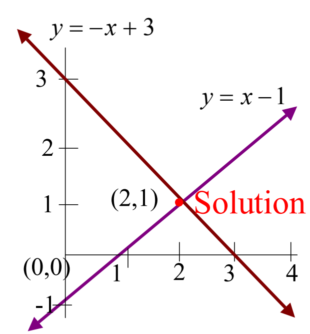
1. Multiply the second equation by 2 to eliminate a variable when adding equations.
2. Add equations
3. Solve for one variable
4. Sub. found variable into either eq. to find other variable.

$$y=-x+3$$
$$y=x-1$$

SOLVING Systems of Equations BY SUBSTITUTION & GRAPHING

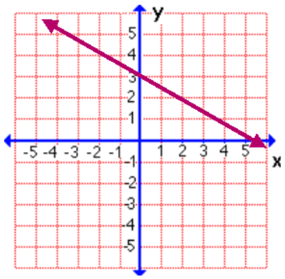
$$-x+3=x-1$$
$$\underline{+x} \quad \underline{+1} \quad \underline{+x} \quad \underline{+1}$$
$$4=2x$$
$$x=2$$
$$y=2-1$$
$$y=1$$

$\boxed{(2,1)}$

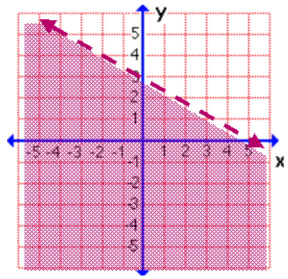


Graph 2-variable inequalities

$y = -1/2\ x + 3$



$y < -1/2\ x + 3$



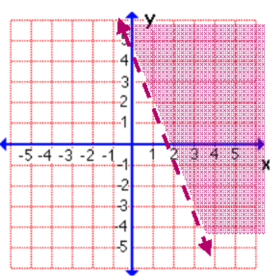
Graph $3y + 6x > 12$
$\underline{-6x} \quad \underline{-6x}$

$3y > -6x + 12$

$\frac{3y}{3} > \frac{-6x}{3} + \frac{12}{3}$

$y > -2x + 4$

$m = -2/1 \quad b = 4$

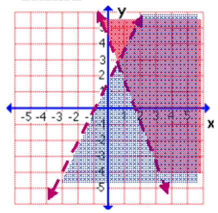


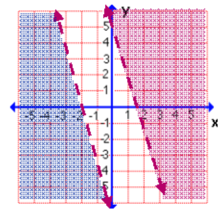
AM Obj 206 Graph 2-Variable Inequalities

1. ***Get in y = mx + b form***
2. *1st point is b(y-intercept)*
3. *Rise & Run(slope m) to 2nd point*
4. *Dashed line if > or <*
   (Solid line if ≥ or ≤)
5. *Shade above if > or ≥(below if < or ≤)*
6. *Test (0,0) for check. If True (0,0) is in the shaded area.*

Solution Possibilities

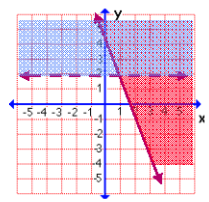
Points in the Double Shaded Area are the Solution

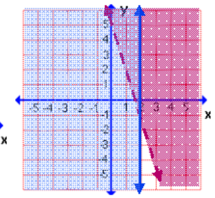




No Double Shade
No Solutions

y>2 & y≥-2x+4





x≥2 & y>2x+4

GRAPH 2-VAR INEQUALITIES

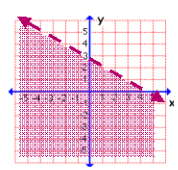
Graph 2-variable inequalities

Notice how the graph of an inequality is a shaded region (or a half-plane).

The region represents all of the ordered pairs that are solutions to the inequality.

The dotted line represents the border of the shaded region.

It is dotted because the line is not included in the solution.



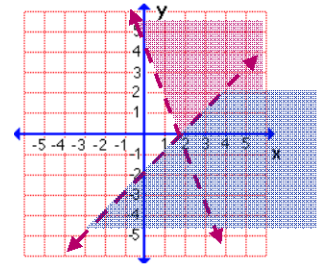
Solve $3y + 6x > 12$
$y<x-2$

$3y > -6x + 12$

$\frac{3y}{3} > \frac{-6x}{3} + \frac{12}{3}$

$y > -2x + 4$

$m = -2/1 \quad b = 4$



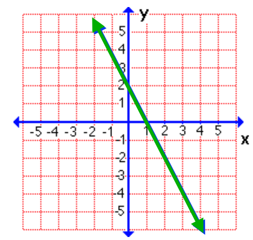
AM Obj 207 Systems of Inequalities

1. ***Get both equations in y = mx + b form & graph***
2. *Dashed line if > or <*
   (Solid line if ≥ or ≤)
3. *Shade above if > or ≥(below if < or ≥)*
4. *Test (0,0) for check or another point*

Solution is the double shaded area.

Example: Solve the system of linear equations by graphing.

$4x + 2y = 4$	$2x + y = 2$
$2y = -4x + 4$	
$y = -2x + 2$	$y = -2x + 2$



Notice that these are the same line (one on top of the other).

Where do they intersect?

Everywhere. The system has **infinite solutions**.

Rules for Graphing Inequalities:

Here are some guidelines that will help you graph linear inequalities:

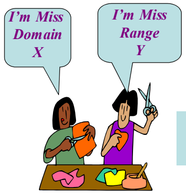
< **or** ≤ shade **below** the line

> **or** ≥ shade **above** the line

Try (0 , 0) and see if it is true

< **or** > the line is not included (**dotted line**)

≤ **or** ≥ **the line is included** (**solid line**)



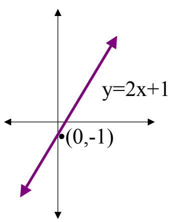
RANGE & DOMAIN VALUES

DOMAIN VALUES	RANGE VALUES

(3, 21)
(5, 35)
(11, 77)

Domain {3,5,11} Range{21,35,77}

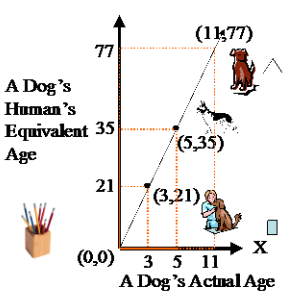
What's the Domain & Range of a Graph of a Straight Line



I'm -□ to +□ (All Real #'s)

I'm also -□ to +□ (All Real #'s)

AM 151 Determining Functions



1. Graph Points

2. Do a vertical line (pencil test)

Or Notice if any x-value has more than one y-value then it's not a function

THIS PASSES TEST SO IT IS A FUNCTION

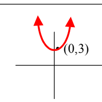
Hi, I'm Domain X and I have To be First

Hi, I'm Range Y and I love to be Second

RELATIONS & FUNCTIONS

Determine if eq. Is a function & Domain and Range.

$$\{(x, y) \mid y = x^2 + 3\}$$



The domain is all real # 's
The range is y □3

Determining Functions & DOMAIN & RANGE

1. Graph the equation
2. Do the pencil test

Note: All second degree equations are parabolas

$y=x^2$ are up & down

$x=y^2$ sideways parabolas

FUN FUNCTION MACHINE

Input x → Function → Output y

Input 5 → f(x)=7x → Output *f(5)= 7•5 = 35*

Read f(x) as the function of x.

f(x) = y = Output = Range value for this x

Outputs

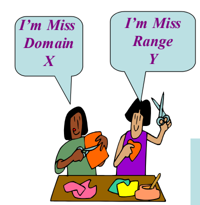
Answers

Inputs x={3,5,11} → f(x)=7x → Outputs y=?
(3, 21), (5,35), (11,77)

Input g={1,4,9} → f(g)=g+5 → Outputs ?
(1, 6), (4,9), (9,14)

Inputs x={1,5,-8} → f(x)=7x-2 → Outputs y=?
(1, 5), (5,33), (-8,-58)

Input h={1,-4,7} → f(h)=2h-1 → Outputs ?
(1,1), (-4,-9), (7,13)



Final Review

Inputs or X-Values or The Domain or 1st Coordinate		Outputs or Y-Values or The Range Or 2nd Coordinate
	(3, 21) (5, 35) (11, 77)	

A relationship is any (x,y) pair or pairs.
A function is a relationship where each input value has only one output value.

Simplifying Square Roots

$$\sqrt{200x^3} =$$

$$= \sqrt{100x^2}\sqrt{2x}$$

$$= 10x\sqrt{2x}$$

1. Factor any perfect squares (4, 9, 16, 25, x^2, y^6…)

(& Put perfect squares in first radical and other factor in 2nd)

2. Take square root of first radical

Divide Radicals

$$Simplify: \sqrt{\frac{30x^4}{6x^2}}$$

$$\frac{\sqrt{30x^4}}{\sqrt{6x^2}} = \sqrt{\frac{30x^4}{6x^2}}$$

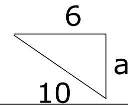
$$= \sqrt{5x^2}$$

$$= \sqrt{x^2}\sqrt{5}$$

$$= x\sqrt{5}$$

1. Divide/Reduce
2. Simplify

Solve using the Pythagorean Theorem



$$a^2 + b^2 = c^2$$

$$a^2 + 6^2 = 10^2$$

$$a^2 + 36 = 100$$

$$a^2 = 64$$

$$a = \sqrt{64} = 8$$

1. Substitute known values into Pythagorean Theorem
2. Solve for unknown variable Simplify radical if needed

QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

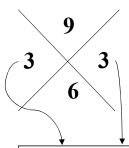
$$\sqrt{x} + \sqrt{x} = ?$$

$$4\sqrt{8} - 3\sqrt{8} = \sqrt{8}$$

$\sqrt{Radicals}$

Square Roots of Perfect Squares

$$\sqrt{x^2 + 6x + 9}$$



$$\boxed{(x+3)(x+3) = (x+3)^2}$$

$$\sqrt{(x+3)^2}$$

$$= x + 3$$

1. Factor any perfect squares
2. Take square root of perfect square.

Visual to REMEMBER

$$\sqrt{}$$

$$\sqrt{PS} \cdot \sqrt{OF}$$

Perfect Square times ***Other Factor***

Perfect squares

1, 4, 9, 16, …

$3x^{25}$

$3x^{25}$ **$9x^{50}$**

EVEN POWERED EXPONENTS ARE SQUARES

Divide Radicals Rationalizing the Denominator

$$\frac{\sqrt{2}}{\sqrt{3}}$$

$$\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{6}}{\sqrt{9}}$$

$$= \frac{\sqrt{6}}{3}$$

1. Mult. Numerator & Denom. By Denom. to get a Perfect Square in Denominator
2. Take the square root of the Perfect Square
3. Simplify

Simplifying Negative Square Roots

$$\sqrt{-9} =$$

$$\sqrt{-9} =$$

$$= \sqrt{9}\ \sqrt{-1}$$

$$= 3i$$

1. Factor any perfect squares (4, 9, 16, 25, 36…)

(& Put perfect squares in first radical and other factor in 2nd)

2. Take square root of first radical

3. The square root of -1 is called IMAGINARY

Multiply Radicals

$$\sqrt{6m} \cdot \sqrt{8m}$$

$$\sqrt{6m} \cdot \sqrt{8m}$$

$$= \sqrt{48m^2}$$

$$= \sqrt{16m^2} \cdot \sqrt{3}$$

$$= 4m\sqrt{3}$$

1. Multiply to one radical
2. Simplify
   - Factor any perfect squares (4, 9, 16, x^2, y^6…)
   - Take square root of first radical

SOLVING QUADRATIC EQUATIONS

Solve with Perfect Square Binomial

$$(x-2)^2 = 49$$

$$\sqrt{(x-2)^2} = \sqrt{49}$$

$$x - 2 = \pm 7$$

$$x - 2 = +7 \qquad x - 2 = -7$$

$$x = \boxed{9} \qquad x = \boxed{-5}$$

1. Get Perfect Squares on Both Sides of Equation.
2. Take Square Root of Perfect Squares
3. Solve Positive Number & Neg. Number
4. Check Answers by Putting into Original Eq.

Solve Quadratic Eq. By Factoring

$$x^2 + 5x = 6$$

$$x^2 + 5x = 6$$

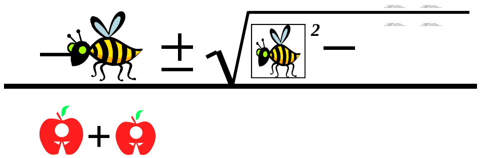
$$x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0$$

$$x + 6 = 0 \quad \& \quad x - 1 = 0$$

$$x = \boxed{-6} \qquad x = \boxed{+1}$$

1. Set Eq = 0
2. Factor
3. Set Each Factor = 0
4. Check Answers by Putting into Original Eq.



$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Linear Vs. Quadratic

Linear	Quadratic
Straight Line	Parabola or "U" Shaped
First Degree Eq.	Second Degree Equation
Exponent is 1	Exponent is 2
ax +by +c = 0	$f(x) = ax^2 + bx + c = 0$
y = mx + b	

Solve with Quadratic Equation

$$x^2 + 2x = 2$$

a=1 b c

$$x^2 + 2x - 2 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(-2)}}{2}$$

$$x = \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{3}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} = \boxed{-1 \pm \sqrt{3}}$$

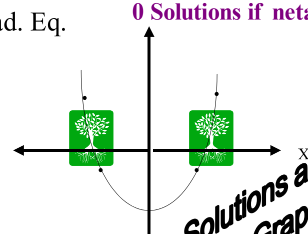
1. Put Equation into Std. Form ($ax^2+bx+c=0$)
2. Plug a, b & c into the Quad. Eq.
   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. Simplify Radical
4. Solve for x

The Discriminant

The Discriminant equals b^2-4ac

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 Solutions if positive
1 Solution if 0
0 Solutions if netative



Solutions are Roots Where Graph crosses the x-axis

Solve by Completing The Square

$$3x^2 + 6x - 3 = 6$$

$$3x^2 + 6x = 9$$

$$x^2 + 2x + \_\_ = 3 + \_\_$$

$$x^2 + 2x + 1 \quad = 3 + 1$$

$$(x+1)^2 = 4$$

$$x + 1 = \pm\sqrt{4}$$

$$x = -1 \pm 2$$

$$x = \boxed{1} \text{ or } x = \boxed{-3}$$

1. First Put c on Right Hand Side of equation.
2. Divide by a
3. Complete The Square
   $(^b/_2)^2 = (^2/_2)^2 = 1$
4. Write Trinomial Square as a Binomial Square
5. Square Root both sides
6. ***Solve for x (Note 2 Answers)***