1 Find the coordinates of the stationary point of each of the following curves:

a $y = x^2 + 4x - 12$

b $y = 2t^3 - 5t^2 - 4t + 13$

2 Let $y = x^4 + x^3 + x^{-2} + 8$.

a Find the average rate of change of y between $x = 1$ and $x = 2$.

b Find the gradient of the curve at $x = 2$.

3 Find the equation of the tangent and the normal to the curve $y = x^3 - 2x^2 + 4$ at the point where $x = 2$.

4 A particle moves from rest in a straight line so that after t seconds it is s metres from a fixed point O on the line, where $s = t^3 - 6t^2 + 12$.

a Find the position of the particle after 3 seconds.

b Find the time and position when the particle comes to rest again.

c Find the total distance travelled in the first 4 seconds.

5 The line $y = ax + 3$ is tangent to the parabola $y = x^2 + x + b$ when $x = 1$. Find the values of the constants a and b.

6 Water is being poured into a tank so that the volume, V mL, of water in the tank at time t minutes is given by $V(t) = \frac{1}{3}\left(8t^2 - \frac{t^3}{2}\right), 0 \le t \le 10.$

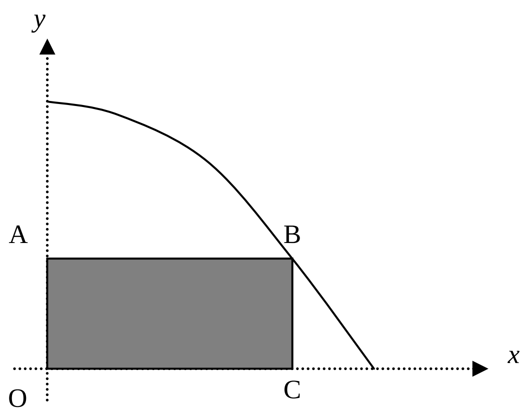
a Find the volume of water in the tank at time:

i $t = 0$ **ii** $t = 10$

b Find the rate of flow of water into the tank at any time t.

c Find the rate of flow of water into the tank at $t = 5$.

7 The diagram shows the graph of $y = 4 - 3x^2$, for $x \geq 0$ and $y \geq 0$. B is a point on the graph and $OABC$ is a rectangle. Find the value of x for which the area of $OABC$ is a maximum.



8 Consider the function $y = x^3 - x^2 - 5x - 3$.

a Factorise and find the x- and y-intercepts for the graph.

b Find the coordinates of the turning points.

c Sketch the graph of the function, labelling all the important features.

9 The derivative of a curve is $\frac{dy}{dx} = -2x + 8$. Find the equation of the tangent to this curve at the point (–2, 3).

10 A rectangular box, made of thin sheet metal and without a lid, is of length $2x$ cm, width x cm and height h cm.

a Write down, in terms of x and h, the area of sheet metal required to make the box.

b Given that the area of sheet metal is 600 cm^2, show that $h = \frac{600 - 2x^2}{6x}$.

c Hence show that the volume, V cm^3, of the box is given by $V = 200x - \frac{2x^3}{3}$.

d Find $\frac{dV}{dx}$ and find the value of x for which V is a maximum.

e Hence calculate the volume of the largest such box that can be constructed using 600 cm^2 of sheet metal.

11 A particle moves in a straight line so that its position x cm relative to O at time t seconds is given by $x = t^2 - 9t + 8$, $t \geq 0$.

a Find its initial velocity.

b Find when and where its velocity equals zero.

c Determine its average velocity for the first 4 seconds.

d Determine its average speed for the first 4 seconds.

12 **a** Let f:[–3, 5] $\rightarrow R$, $f(x) = x^2 + 2$. Find the maximum and minimum value of the function.

b Let f:[–2, 1] $\rightarrow R$, $f(x) = x^3 + 2x + 6$. Find the maximum and minimum value of the function for its domain

Answers

1 **a** (–2, –16) **b** $\left(-\frac{1}{3}, \frac{370}{27}\right)$ and (2, 1)

2 **a** 21·25 **b** 43·75

3 Tangent: $y = 4x - 4$; normal: $4y + x = 18$

4 **a** –15 m **b** $t = 4$ s, $s = -20$

c 32 m

5 $a = 3$ and $b = 4$

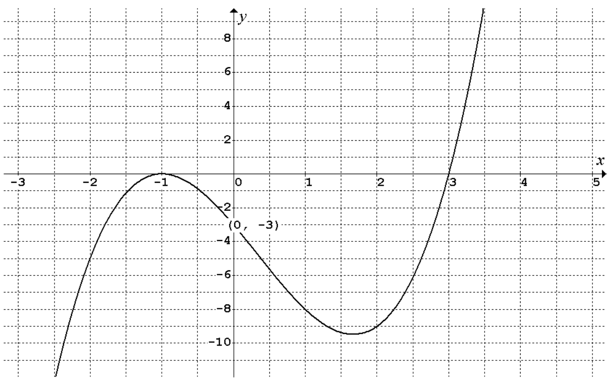
6 **a** **i** 0 **ii** 100

b $V'(t) = \frac{1}{3}\left(16t - \frac{3t^2}{2}\right)$ **c** $14\frac{1}{6}$ mL/min

7 $x = \frac{2}{3}$

8 **a** $(x + 1)^2(x - 3)$ **b** (–1, 0) and $\left(\frac{5}{3}, -\frac{256}{27}\right)$

c



9 $y = 12x + 27$

10 **a** Area = $2x^2 + 6xh$

d $\frac{dV}{dx} = 200 - 2x^2$, V is maximum for $x = 10$

e Max volume = $\frac{4000}{3}$ cm^3

11 **a** –9 cm/s **b** $t = \frac{9}{2}$, $x = -\frac{49}{4}$

c –5 cm/s **d** $\frac{25}{8}$ cm/s

12 **a** Maximum value = 27; minimum value = 2

b Minimum value = –6; maximum value = 9