Chapter 4: Quadratics

Exercise 4A Solutions

1

a $2(x - 4) = 2x - 8$

b $-2(x - 4) = -2x + 8$

c $3(2x - 4) = 6x - 12$

d $-3(4 - 2x) = 6x - 12$

e $x(x - 1) = x^2 - x$

f $2x(x - 5) = 2x^2 - 10x$

2

a $(2x + 4x) + 1 = 6x + 1$

b $(2x + x) - 6 = 3x - 6$

c $(3x - 2x) + 1 = x + 1$

d $(-x + 2x + 4x) - 3 = 5x - 3$

3

a $8(2x - 3) - 2(x + 4)$
$= 16x - 24 - 2x - 8$
$= 14x - 32$

b $2x(x - 4) - 3x$
$= 2x^2 - 8x - 3x$
$= 2x^2 - 11x$

c $4(2 - 3x) + 4(6 - x)$
$= 8 - 12x + 24 - 4x$
$= 32 - 16x$

d $4 - 3(5 - 2x)$
$= 4 - 15 + 6x$
$= 6x - 11$

4

a $2x(x - 4) - 3x$
$= 2x^2 - 8x - 3x$
$= 2x^2 - 11x$

b $2x(x - 5) + x(x - 5)$
$= 2x^2 - 10x + x^2 - 5x$
$= 3x^2 - 15x$

c $2x(-10 - 3x)$
$= -20x - 6x^2$

d $3x(2 - 3x + 2x^2)$
$= 6x - 9x^2 + 6x^3$

e $3x - 2x(2 - x)$
$= 3x - 4x + 2x^2$
$= 2x^2 - x$

f $3(4x - 2) - 6x$
$= 12x - 6 - 6x$
$= 6x - 6$

5

a $(3x - 7)(2x + 4)$
$= 6x^2 + 12x - 14x - 28$
$= 6x^2 - 2x - 28$

b $(x - 10)(x - 12)$
$= x^2 - 10x - 12x + 120$
$= x^2 - 22x + 120$

c $(3x - 1)(12x + 4)$
$= 36x^2 + 12x - 12x - 4$
$= 36x^2 - 4$

d $(4x - 5)(2x - 3)$
$= 8x^2 - 12x - 10x + 15$
$= 8x^2 - 22x + 15$

e $(x - \sqrt{3})(x - 2)$
$= x^2 - 2x - \sqrt{3}x + 2\sqrt{3}$
$= x^2 - (2 + \sqrt{3})x + 2\sqrt{3}$

f $(2x - \sqrt{5})(x + \sqrt{5})$
$= 2x^2 + 2\sqrt{5}x - \sqrt{5}x - 5$
$= 2x^2 + \sqrt{5}x - 5$

6

a $(x-4)^2$
$= x^2 - 4x - 4x + 16$
$= x^2 - 8x + 16$

b $(2x-3)^2$
$= 4x^2 - 6x - 6x + 9$
$= 4x^2 - 12x + 9$

c $(6-2x)^2$
$= 36 - 12x - 12x + 4x^2$
$= 36 - 24x + 4x^2$

d $\left(x - \frac{1}{2}\right)^2$
$= x^2 - \frac{x}{2} - \frac{x}{2} + \frac{1}{4}$
$= x^2 - x + \frac{1}{4}$

e $(x - \sqrt{5})^2$
$= x^2 - \sqrt{5}\,x - \sqrt{5}\,x + 5$
$= x^2 - 2\sqrt{5}\,x + 5$

f $(x - 2\sqrt{3})^2$
$= x^2 - 2\sqrt{3}\,x - 2\sqrt{3}\,x + 4(3)$
$= x^2 - 4\sqrt{3}\,x + 12$

7

a $(2x-3)(3x^2+2x-4)$
$= 6x^3 + 4x^2 - 8x - 9x^2 - 6x + 12$
$= 6x^3 - 5x^2 - 14x + 12$

b $(x-1)(x^2+x+1)$
$= x^3 + x^2 + x - x^2 - x - 1$
$= x^3 - 1$

c $(6 - 2x - 3x^2)(4 - 2x)$
$= 24 - 12x - 8x + 4x^2 - 12x^2 + 6x^3$
$= 24 - 20x - 8x^2 + 6x^3$

d $(x-3)(x+3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$

e $(2x-4)(2x+4)$
$= 4x^2 + 8x - 8x - 16$
$= 4x^2 - 16$

f $(9x-11)(9x+11)$
$= 81x^2 + 99x - 99x + 121$
$= 81x^2 - 121$

g $(5x-3)(x+2) - (2x-3)(x+3)$
$= (5x^2 + 10x - 3x - 6) - (2x^2 + 6x - 3x - 9)$
$= (5x^2 + 7x - 6) - (2x^2 + 3x - 9)$
$= 3x^2 + 4x + 3$

h $(2x+3)(3x-2) - (4x+2)(4x-2)$
$= (6x^2 - 4x + 9x - 6) - (16x^2 - 8x + 8x - 4)$
$= (6x^2 + 5x - 6) - (16x^2 - 4)$
$= -10x^2 + 5x - 2$

i $(x - y + z)(x - y - z)$
$= [(x-y) + z][(x-y) - z]$
$= (x-y)^2 - z^2$
$= x^2 - 2xy + y^2 - z^2$

j $(x-y)(a-b)$
$= ax - bx - ay + by$

8

a **i** $A = x^2 + 2x + 1$
ii $A = (x+1)^2$

b **i** $A = (x-1)^2 + 2(x-1) + 1$
ii $A = x^2$

Exercise 4B Solutions

1

a $2x + 4 = 2(x + 2)$

b $4a - 8 = 4(a - 2)$

c $6 - 3x = 3(2 - x)$

d $2x - 10 = 2(x - 5)$

e $18x + 12 = 6(3x + 2)$

f $24 - 16x = 8(3 - 2x)$

2

a $4x^2 - 2xy = 2x(2x - y)$

b $8ax + 32xy = 8x(a + 4y)$

c $6ab - 12b = 6b(a - 2)$

d $6xy + 14x^2y = 2xy(3 + 7x)$

e $x^2 + 2x = x(x + 2)$

f $5x^2 - 15x = 5x(x - 3)$

g $-4x^2 - 16x = -4x(x + 4)$

h $7x + 49x^2 = 7x(1 + 7x)$

i $2x - x^2 = x(2 - x)$

j $6x^2 - 9x = 3x(2x - 3)$

k $7x^2y - 6y^2x = xy(7x - 6y)$

l $8x^2y^2 + 6y^2x = 2xy^2(4x + 3)$

3

a $x^3 + 5x^2 + x + 5$
$= x^2(x + 5) + (x + 5)$
$= (x + 5)(x^2 + 1)$

b $x^2y^2 - x^2 - y^2 + 1$
$= x^2(y^2 - 1) - (y^2 - 1)$
$= (x^2 - 1)(y^2 - 1)$
$= (x - 1)(x + 1)(y - 1)(y + 1)$

c $ax + ay + bx + by$
$= a(x + y) + b(x + y)$
$= (a + b)(x + y)$

d $a^3 - 3a^2 + a - 3$
$= a^2(a - 3) + (a - 3)$
$= (a^2 + 1)(a - 3)$

e $x^3 - bx^2 - a^2x + a^2b$
$= x^2(x - b) - a^2(x - b)$
$= (x^2 - a^2)(x - b)$
$= (x - a)(x + a)(x - b)$

4

a $x^2 - 36 = (x - 6)(x + 6)$

b $4x^2 - 81 = (2x - 9)(2x + 9)$

c $2x^2 - 98 = 2(x^2 - 49)$
$= 2(x - 7)(x + 7)$

d $3ax^2 - 27a = 3a(x^2 - 9)$
$= 3a(x - 3)(x + 3)$

e $(x - 2)^2 - 16$
$= (x - 2 - 4)(x - 2 + 4)$
$= (x - 6)(x + 2)$

f $25 - (2 + x)^2$
$= (5 - (2 + x))(5 + (2 + x))$
$= (3 - x)(7 + x)$

g $3(x + 1)^2 - 12 = 3((x + 1)^2 - 4)$
$= 3(x + 1 - 2)(x + 1 + 2)$
$= 3(x - 1)(x + 3)$

h $(x - 2)^2 - (x + 3)^2$
$= ((x - 2) - (x + 3))((x - 2) + (x + 3))$
$= (x - 2 - x - 3)(x - 2 + x + 3)$
$= -5(2x + 1)$

5

a Check signs: must be + and –

$x^2 - 7x - 18 = (x - 9)(x + 2)$

b Check signs: must be – and –

$y^2 - 19y + 48 = (y - 16)(y - 3)$

c Check signs: must be – and –

$3x^2 - 7x + 2 = (3x - a)(x - b)$

$a + 3b = 7$; $ab = 2$

$b = 2$, $a = 1$:

$3x^2 - 7x + 2 = (3x - 1)(x - 2)$

d Check signs: must be + and +

$6x^2 + 7x + 2 = (6x + a)(x + b)$

$a + 6b = 7$, $ab = 2$; no solution.

Try:

$6x^2 + 7x + 2 = (3x + a)(2x + b)$

$2a + 3b = 7$, $ab = 2$

$a = 2$, $b = 1$

$6x^2 + 7x + 2 = (3x + 2)(2x + 1)$

e $a^2 - 14a + 24 = (a - 12)(a - 2)$

f $a^2 + 18a + 81 = (a + 9)(a + 9)$

$= (a + 9)^2$

g $5x^2 + 23x + 12 = (5x + a)(x + b)$

$a + 5b = 23$; $ab = 12$

$\therefore\ b = 4$, $a = 3$

$5x^2 + 23x + 12 = (5x + 3)(x + 4)$

h Check signs: must be + and –

$3y^2 - 12y - 36 = 3(y^2 - 4y - 12)$

$= 3(y^2 - 4y - 12)$

$= 3(y + a)(y - b)$

$a - b = -4$; $ab = 12$

$\therefore\ a = 2$, $b = 6$

$3y^2 - 12y - 36 = 3(y + 2)(y - 6)$

i $2x^2 - 18x + 28 = 2(x^2 - 9x + 14)$

$= 2(x - 2)(x - 7)$

j $4x^2 - 36x + 72 = 4(x^2 - 9x + 18)$

$= 4(x - 6)(x - 3)$

k $3x^2 + 15x + 18 = 3(x^2 + 5x + 6)$

$= 3(x + 3)(x + 2)$

l $ax^2 + 7ax + 12a = a(x^2 + 7x + 12)$

$= a(x + 3)(x + 4)$

m $5x^3 - 16x^2 + 12x = x(5x^2 - 16x + 12)$

$= x(5x - a)(x - b)$

$a + 5b = 16$; $ab = 12$

$\therefore\ a = 6$, $b = 2$

$5x^3 - 16x^2 + 12x = x(5x - 6)(x - 2)$

n $48x - 24x^2 + 3x^3 = 3x(16 - 8x + x^2)$

$= 3x(4 - x)^2$ or $3x(x - 4)^2$

o $(x - 1)^2 + 4(x - 1) + 3$

Put $y = x - 1$:

$= y^2 + 4y + 3$

$= (y + 3)(y + 1)$

$= (x - 1 + 3)(x - 1 + 1)$

$= x(x + 2)$

Exercise 4C Solutions

1

a $(x-2)(x-3)=0,\ \therefore x=2,\ 3$

b $x(2x-4)=0,\ \therefore 2x(x-2)=0$

$\therefore x=0,\ 2$

c $(x-4)(2x-6)=0$

$\therefore 2(x-4)(x-3)=0$

$\therefore x=3,\ 4$

d $(3-x)(x-4)=0$

$\therefore x=3,\ 4$

e $(2x-6)(x+4)=0$

$\therefore 2(x-3)(x+4)=0$

$\therefore x=3,\ -4$

f $2x(x-1)=0,\ \therefore x=0,\ 1$

g $(5-2x)(6-x)=0$

$\therefore 2\left(\frac{5}{2}-x\right)(6-x)=0$

$\therefore x=\frac{5}{2},\ 6$

h $x^2=16,\ \therefore x^2-16=0$

$\therefore (x-4)(x+4)=0$

$\therefore x=4,\ -4$

2

a $x^2-4x-3=0$

$\therefore x=-0.65,\ 4.65$

b $2x^2-4x-3=0$

$\therefore x=-0.58,\ 2.58$

c $-2x^2-4x+3=0$

$\therefore x=-2.58,\ 0.58$

3

a $x^2-6x+8=0$

$\therefore (x-2)(x-4)=0$

$\therefore x=2,\ 4$

b Check signs: must be + and –

$x^2-8x-33=0$

$\therefore (x-a)(x+b)=0$

$a-b=8;\ ab=33$

$a=11;\ b=3$

$(x-11)(x+3)=0$

$\therefore x=11,\ -3$

c $x(x+12)=64$

$x^2+12x-64=0$

Check signs: must be + and –

$\therefore (x-a)(x+b)=0$

$b-a=12;\ ab=64;$

$b=16;\ a=4$

$(x-4)(x+16)=0$

$\therefore x=4,\ -16$

d Check signs: must be + and –

$x^2+5x-14=0$

$(x-a)(x+b)=0$

$b-a=5;\ ab=14;$

$b=7;\ a=2$

$(x-2)(x+7)=0$

$\therefore x=2,\ -7$

e $2x^2+5x+3=0$

$\therefore (2x+a)(x+b)=0$

$a+2b=5;\ ab=3$

$a=3;\ b=2$

$(2x+3)(x+1)=0$

$\therefore x=-\frac{3}{2},\ -1$

f $4x^2-8x+3=0$

$\therefore (2x-a)(2x-b)=0$

$2a+2b=8;\ ab=3$

$a=3;\ b=1$

$(2x-3)(2x-1)=0$

$\therefore x=\frac{3}{2},\ \frac{1}{2}$

g $x^2 = 5x + 24$, $\therefore\ x^2 - 5x - 24 = 0$

Check signs: must be + and –

$\therefore\ (x - a)(x + b) = 0$

$a - b = 5$; $ab = 24$

$a = 8$; $b = 3$

$(x - 8)(x + 3) = 0$

$\therefore\ x = 8, -3$

h $6x^2 + 13x + 6 = 0$

$\therefore\ (3x + a)(2x + b) = 0$

$2a + 3b = 13$; $ab = 6$

$a = 2$; $b = 3$

$(3x + 2)(2x + 3) = 0$

$\therefore\ x = -\frac{2}{3}, -\frac{3}{2}$

i $2x^2 - x = 6$

$\therefore\ 2x^2 - x - 6 = 0$

$\therefore\ x = -\frac{3}{2}, 2$

j $6x^2 + 15 = 23x$

$\therefore\ 6x^2 - 23x + 15 = 0$

$\therefore\ (6x - a)(x - b) = 0$

$a + 6b = 23$; $ab = 15$

$b = 3$; $a = 5$

$(6x - 5)(x - 3) = 0$

$\therefore\ x = \frac{5}{6}, 3$

k Check signs: must be + and –

$2x^2 - 3x - 9 = 0$

$\therefore\ (2x - a)(x + b) = 0$

$2b - a = -3$; $ab = 9$

$b = -3$; $a = -3$

$(2x + 3)(x - 3) = 0$

$\therefore\ x = -\frac{2}{3}, 3$

l $10x^2 - 11x + 3 = 0$

$\therefore\ (5x - a)(2x - b) = 0$

$2a + 5b = 11$; $ab = 3$

$a = 3$; $b = 1$

$(5x - 3)(2x - 1) = 0$

$\therefore\ x = \frac{3}{5}, \frac{1}{2}$

m $12x^2 + x = 6$

$\therefore\ 12x^2 + x - 6 = 0$

Check signs: must be + and –

$\therefore\ (6x - a)(2x + b) = 0$

$6b - 2a = 1$; $ab = 6$; no solution

$\therefore\ (4x - a)(3x + b) = 0$

$4b - 3a = 1$; $ab = 6$

$a = -3$; $b = -2$

$(4x + 3)(3x - 2) = 0$

$\therefore\ x = -\frac{3}{4}, \frac{2}{3}$

n $4x^2 + 1 = 4x$

$\therefore\ 4x^2 - 4x + 1 = 0$

$\therefore\ (2x - 1)^2 = 0$, $\therefore\ x = \frac{1}{2}$

o $x(x + 4) = 5$

$\therefore\ x^2 + 4x - 5 = 0$

Check signs: must be + and –

$\therefore\ (x - a)(x + b) = 0$

$b - a = 4$; $ab = 5$

$b = 5$; $a = 1$

$(x - 1)(x + 5) = 0$

$\therefore\ x = 1, -5$

p $\frac{1}{7}x^2 = \frac{3}{7}x$

$\therefore\ x^2 = 3x$, $\therefore\ x^2 - 3x = 0$

$\therefore\ x(x - 3) = 0$, $\therefore\ x = 0, 3$

q $x^2 + 8x = -15$

$x^2 + 8x + 15 = 0$

$(x + 5)(x + 3) = 0$

$\therefore\ x = -5, -3$

r $5x^2 = 11x - 2$

$\therefore\ 5x^2 - 11x + 2 = 0$

$\therefore\ (5x - a)(x - b) = 0$

$a + 5b = 11$; $ab = 2$

$a = 1$; $b = 2$

$(5x - 1)(x - 2) = 0$

$\therefore\ x = \frac{1}{5}, 2$

4 $M = \frac{wl}{2}x - \frac{w}{2}x^2$

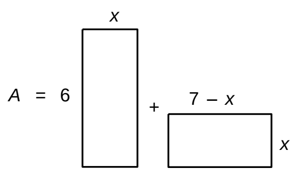
$\therefore 104x - 8x^2 = 288$

$\therefore x^2 - 13x + 36 = 0$

$\therefore (x - 4)(x - 9) = 0$

$\therefore x = 4, 9$

5 Cut vertically down middle:



$\therefore A = 6x + x(7 - x) = 30$

$\therefore 6x + 7x - x^2 = 30$

$\therefore x^2 - 13x - 30 = 0$

$\therefore (x - 3)(x - 10) = 0$

$\therefore x = 3, 10$

However, $0 < x < 7$ so $x = 3$

6 $h = 70t - 16t^2 = 76$

$\therefore 16t^2 - 70t + 76 = 0$

$\therefore 8t^2 - 35t + 38 = 0$

$\therefore (8t - 19)(t - 2) = 0$

$\therefore t = 2, \frac{19}{8}$ seconds

7 $D = \frac{n}{2}(n - 3) = 65$

$\therefore n^2 - 3n - 130 = 0$

$\therefore (n - a)(n + b) = 0$

$b - a = -3$; $ab = 130$

$b = 10$; $a = 13$

$(n - 10)(n + 13) = 0$

$\therefore n = -10, 13$

Since $n > 0$, the polygon has 13 sides.

8 $R = 1.6 + 0.03v + 0.003v^2 = 10.6$

$\therefore 3v^2 + 1600 + 30v = 10600$

$\therefore 3v^2 + 30v - 9000 = 0$

$\therefore v^2 + 10v - 3000 = 0$

$\therefore (v - a)(v + b) = 0$

$b - a = 10$; $ab = 3000$

$b = 60$, $a = 50$

$(v - 50)(v + 60) = 0$

$\therefore v = 50, -60$

$v \geq 0, \therefore v = 50$ km/h

9 $P = 2L + 2W = 16$

$\therefore L = 8 - W$

$A = LW = W(8 - W) = 12$

$\therefore 8W - W^2 = 12$

$\therefore W^2 - 8W + 12 = 0$

$\therefore (W - 2)(W - 6) = 0$

$\therefore W = 2, 6$

Length = 6 cm, width = 2 cm

10

$A = \frac{bh}{2} = 15$

$h = b - 1, \therefore A = \frac{b}{2}(b - 1)$

$\therefore \frac{b}{2}(b - 1) = 15$

$\therefore b^2 - b = 30, \therefore b^2 - b - 30 = 0$

$\therefore (b + 5)(b - 6) = 0$

$\therefore b = 6, -5$

$b \geq 0, \therefore b = 6\text{cm}$

Therefore height (altitude) = 5 cm

11 $e = c + 30 \ldots (1)$

$\frac{1800}{e} + 10 = \frac{1800}{c} \ldots (2)$

Substitute (1) into (2):

$\frac{1800}{c+30} + 10 = \frac{1800}{c}$

$\therefore\ 1800c + 10c(c+30) = 1800(c+30)$

$\therefore\ 1800c + 10c^2 + 300c = 1800c + 54000$

$\therefore\ 10c^2 + 300c = 54000$

$\therefore\ c^2 + 30c - 5400 = 0$

$\therefore\ (c-a)(c+b) = 0$

$b - a = 30$;

$ab = 5400$

$b = 90$, $a = 60$

$(c-60)(c+90) = 0$

$\therefore\ c = \$60$

Cheap seats are \$60, expensive \$90

12 Original cost per person = x

Original members = N where $Nx = 2100$

$\therefore\ x = \frac{2100}{N}$

Later: $(N-7)(x+10) = 2100$

$\therefore\ (N-7)\left(\frac{2100}{N} + 10\right) = 2100$

$\therefore\ (N-7)(2100 + 10N) = 2100N$

$\therefore\ 2100N - 14700 + 10N^2 - 70N = 2100N$

$\therefore\ -14700 + 10N^2 - 70N = 0$

$\therefore\ N^2 - 7N - 1470 = 0$

$\therefore\ (N-a)(N+b) = 0$

$a — b = 7$; $ab = 1470$

$a = 42$; $b = 35$

$\therefore\ (N-42)(N+35) = 0$

Since $N > 7$, $N = 42$

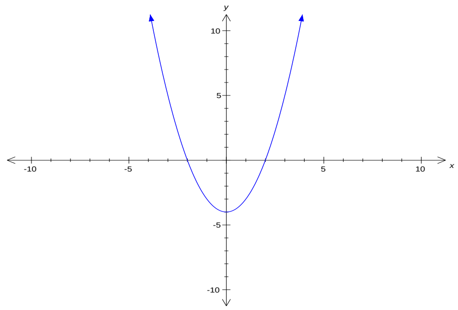
So 42 members originally agreed to go on the bus.

Exercise 4D Solutions

1

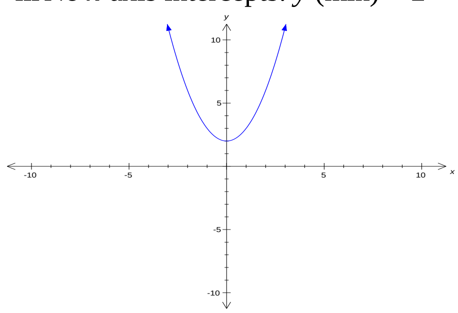
a $y = x^2 - 4$

i turning point at (0, –4)
ii the axis of symmetry $x = 0$
iii the x-axis intercepts (–2,0) and (2,0)



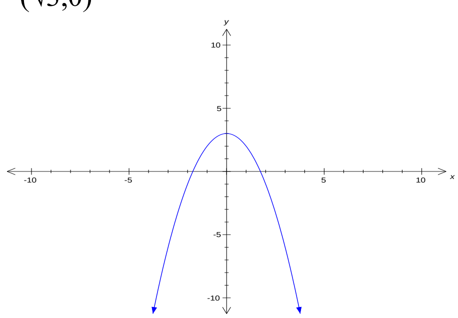
b $y = x^2 + 2$

i turning point at (0,2)
ii the axis of symmetry $x = 0$
iii No x-axis intercepts: y (min) = 2



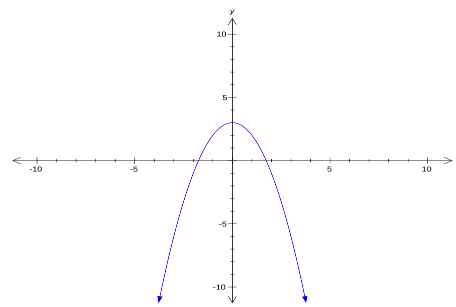
c $y = -x^2 + 3$

i turning point at (0,3)
ii the axis of symmetry $x = 0$
iii the x-axis intercepts $(-\sqrt{3},0)$ and $(\sqrt{3},0)$



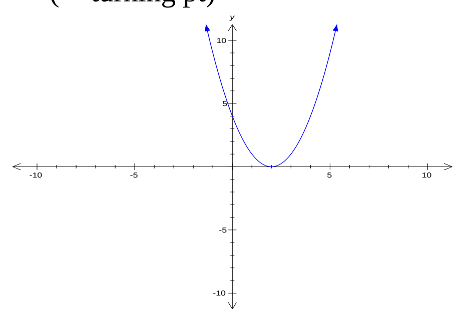
d $y = -2x^2 + 5$

i turning point at (0,5)
ii the axis of symmetry $x = 0$
iii the x-axis intercepts $\left(-\sqrt{\frac{5}{2}},0\right)$ and $\left(\sqrt{\frac{5}{2}},0\right)$



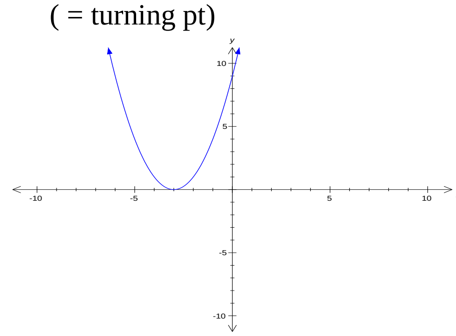
e $y = (x - 2)^2$

i turning point at (2,0)
ii the axis of symmetry $x = 2$
iii the x-axis intercept (2,0) (= turning pt)



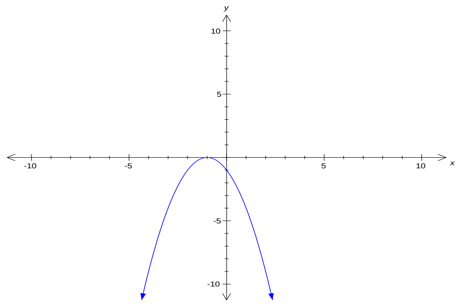
f $y = (x + 3)^2$

i turning point at (–3,0)
ii the axis of symmetry $x = -3$
iii the x-axis intercept (–3,0) (= turning pt)



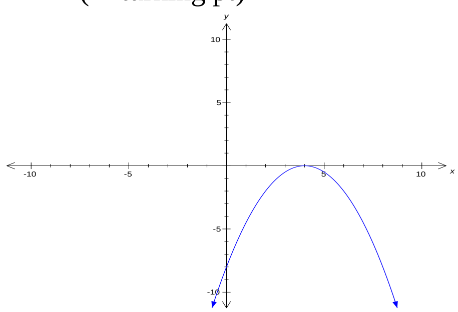
g $y = -(x+1)^2$

i turning point at (–1,0)
ii the axis of symmetry $x = -1$
iii the x-axis intercept (–1,0)
(= turning pt)



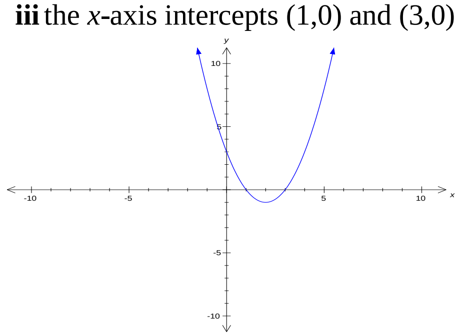
h $y = -\frac{1}{2}(x-4)^2$

i turning point at (4,0)
ii the axis of symmetry $x = 4$
iii the x-axis intercept (4,0)
(= turning pt)



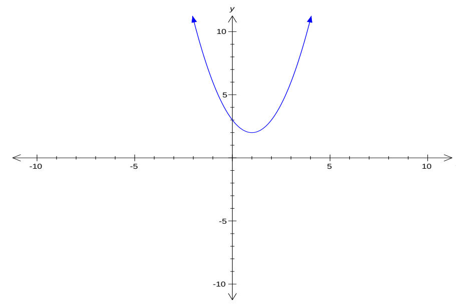
i $y = (x-2)^2 - 1$

i turning point at (2,–1)
ii the axis of symmetry $x = 2$
iii the x-axis intercepts (1,0) and (3,0)



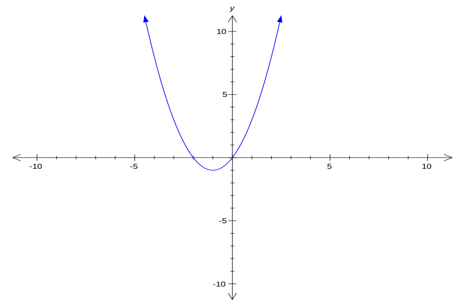
j $y = (x-1)^2 + 2$

i turning point at (1,2)
ii the axis of symmetry $x = 1$
iii no x-axis intercepts



k $y = (x+1)^2 - 1$

i turning point at (–1,–1)
ii the axis of symmetry $x = -1$
iii the x–axis intercepts (0,0) and (–2,0)

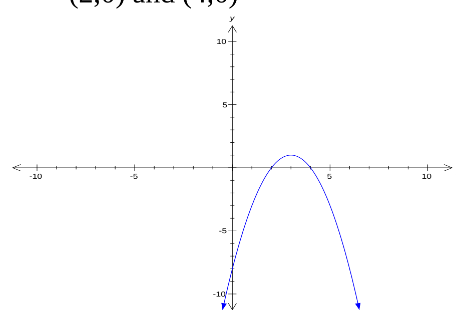


l $y = -(x-3)^2 + 1$

i turning point at (3,1)
ii the axis of symmetry $x = 3$
iii the x–axis intercepts:

$$y = -(x-3)^2 + 1 = 0$$
$$\therefore \quad (x-3)^2 = 1$$
$$\therefore \quad x - 3 = \pm 1$$
$$\therefore \quad x = 3 \pm 1$$

(2,0) and (4,0)

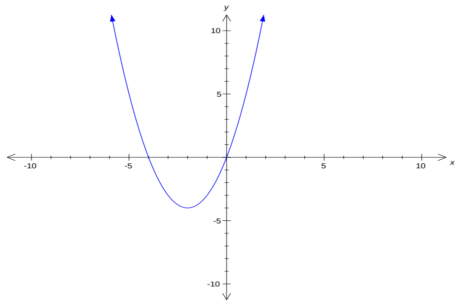


m $y = (x+2)^2 - 4$

i turning point at (–2,–4)

ii the axis of symmetry $x = -2$

iii the x–axis intercepts (0,0) and (–4,0)



n $y = 2(x+2)^2 - 18$

i turning point at (–2,–18)

ii the axis of symmetry $x = -2$

iii the x–axis intercepts:

$$y = \ y = 2(x+2)^2 - 18 = 0$$

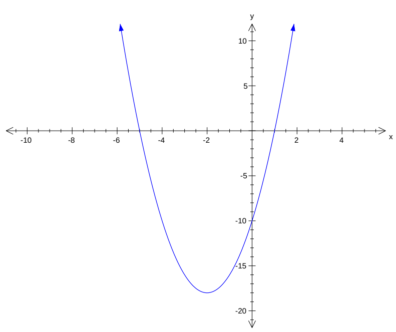
$$\therefore \quad 2(x+2)^2 = 18$$

$$\therefore \quad (x+2)^2 = 9$$

$$\therefore \quad x + 2 = \pm 3$$

$$\therefore \quad x = -2 \pm 3$$

(–5,0) and (1,0)



o $y = -3(x-4)^2 + 3$

i turning point at (4,3)

ii the axis of symmetry $x = 4$

iii the x–axis intercepts:

$$y = \quad -3(x-4)^2 + 3 = 0$$

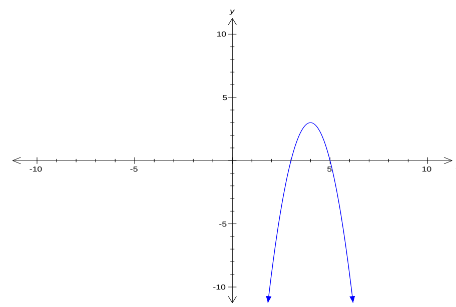
$$\therefore \quad 3(x-4)^2 = 3$$

$$\therefore \quad (x-4)^2 = 1$$

$$\therefore \quad x - 4 = \pm 1$$

$$\therefore \quad x = 4 \pm 1$$

(5,0) and (3,0)



p $y = -\frac{1}{2}(x+5)^2 - 2$

i turning point at (–5,–2)

ii the axis of symmetry $x = -5$

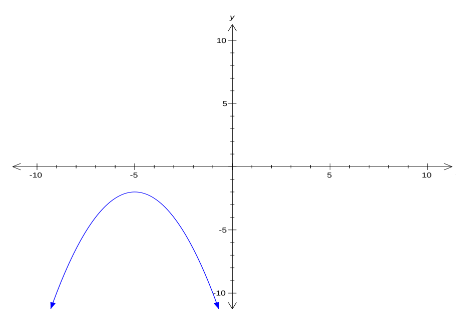
iii the x–axis intercepts:

$$y = \quad -\frac{1}{2}(x+5)^2 - 2 = 0$$

$$\therefore \quad -\frac{1}{2}(x+5)^2 = 2$$

$$\therefore \quad (x+5)^2 = -4$$

No x–axis intercepts because no real roots.



q $y = 3(x+2)^2 - 12$

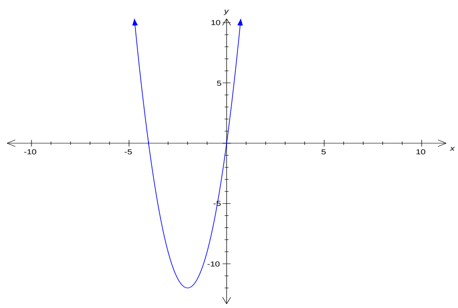
i turning point at (–2,–12)

ii the axis of symmetry $x = -2$

iii the x–axis intercepts:

$$y = 3(x+2)^2 - 12 = 0$$
$$\therefore \quad 3(x+2)^2 = 12$$
$$\therefore \quad (x+2)^2 = 4$$
$$\therefore \quad x+2 = \pm 2$$
$$\therefore \quad x = -2 \pm 2$$

(0,0) and (–4,0)



r $y = -4(x-2)^2 + 8$

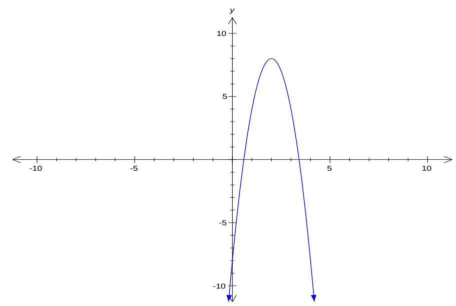
i turning point at (2,8)

ii the axis of symmetry $x = 2$

iii the x–axis intercepts:

$$y = -4(x-2)^2 + 8 = 0$$
$$\therefore \quad 4(x-2)^2 = 8$$
$$\therefore \quad (x-2)^2 = 2$$
$$\therefore \quad x-2 = \pm\sqrt{2}$$
$$\therefore \quad x = 2 \pm \sqrt{2}$$

$(2-\sqrt{2}, 0)$ and $(2+\sqrt{2}, 0)$



Exercise 4E Solutions

1

a $(x-1)^2 = x^2 - 2x + 1$

b $(x+2)^2 = x^2 + 4x + 4$

c $(x-3)^2 = x^2 - 6x + 9$

d $(-x+3)^2 = x^2 - 6x + 9$

e $(-x-2)^2 = (-1)^2(x+2)^2$
$= x^2 + 4x + 4$

f $(x-5)^2 = x^2 - 10x + 25$

g $\left(x-\frac{1}{2}\right)^2 = x^2 - x + \frac{1}{4}$

h $\left(x-\frac{3}{2}\right)^2 = x^2 - 3x + \frac{9}{4}$

2

a $x^2 - 4x + 4 = (x-2)^2$

b $x^2 - 12x + 36 = (x-6)^2$

c $-x^2 + 4x - 4 = -(x^2 - 4x + 4)$
$= -(x-2)^2$

d $2x^2 - 8x + 8 = 2(x^2 - 4x + 4)$
$= 2(x-2)^2$

e $-2x^2 + 12x - 18$
$= -2(x^2 - 6x + 9)$
$= -2(x-3)^2$

f $x^2 - x + \frac{1}{4} = \left(x-\frac{1}{2}\right)^2$

g $x^2 - 3x + \frac{9}{4} = \left(x-\frac{3}{2}\right)^2$

h $x^2 + 5x + \frac{25}{4} = \left(x+\frac{5}{2}\right)^2$

3

a $x^2 - 2x - 1 = 0$

$\therefore x^2 - 2x + 1 - 2 = 0$

$\therefore (x-1)^2 - 2 = 0$

$\therefore (x-1)^2 = 2$

$\therefore x - 1 = \pm\sqrt{2}$

$\therefore x = 1 \pm \sqrt{2}$

b $x^2 - 4x - 2 = 0$

$\therefore x^2 - 4x + 4 - 6 = 0$

$\therefore (x-2)^2 - 6 = 0$

$\therefore (x-2)^2 = 6$

$\therefore x - 2 = \pm\sqrt{6}$

$\therefore x = 2 \pm \sqrt{6}$

c $x^2 - 6x + 2 = 0$

$\therefore x^2 - 6x + 9 - 7 = 0$

$\therefore (x-3)^2 - 7 = 0$

$\therefore (x-3)^2 = 7$

$\therefore x - 3 = \pm\sqrt{7}$

$\therefore x = 3 \pm \sqrt{7}$

d $x^2 - 5x + 2 = 0$

$\therefore x^2 - 5x + \frac{25}{4} + 2 - \frac{25}{4} = 0$

$\therefore \left(x-\frac{5}{2}\right)^2 - \frac{17}{4} = 0$

$\therefore \left(x-\frac{5}{2}\right)^2 = \frac{17}{4}$

$\therefore x - \frac{5}{2} = \pm\frac{1}{2}\sqrt{17}$

$\therefore x = \frac{5 \pm \sqrt{17}}{2}$

e $2x^2 - 4x + 1 = 0$

$\therefore 2\left(x^2 - 2x + \frac{1}{2}\right) = 0$

$\therefore x^2 - 2x + 1 - \frac{1}{2} = 0$

$\therefore (x-1)^2 = \frac{1}{2}$

$\therefore x - 1 = \pm\frac{1}{\sqrt{2}}$

$\therefore x = \frac{2 \pm \sqrt{2}}{2}$

f $3x^2 - 5x - 2 = 0$

$$\therefore \quad 3\left(x^2 - \frac{5x}{3} - \frac{2}{3}\right) = 0$$

$$\therefore \quad x^2 - \frac{5x}{3} - \frac{2}{3} = 0$$

$$\therefore x^2 - \frac{5x}{3} + \frac{25}{36} - \frac{2}{3} - \frac{25}{36} = 0$$

$$\therefore \quad \left(x - \frac{5}{6}\right)^2 - \frac{49}{36} = 0$$

$$\therefore \quad \left(x - \frac{5}{6}\right)^2 = \frac{49}{36}$$

$$\therefore \quad x - \frac{5}{6} = \pm\frac{7}{6}$$

$$\therefore \quad x = \frac{5}{6} \pm \frac{7}{6} = 2, -\frac{1}{3}$$

g $x^2 + 2x + k = 0$

$$\therefore x^2 + 2x + 1 - (1 - k) = 0$$

$$\therefore \quad (x+1)^2 - (1-k) = 0$$

$$\therefore \quad x + 1 = \pm\sqrt{1-k}$$

h $kx^2 + 2x + k = 0$

$$\therefore \quad x^2 + \frac{2x}{k} + 1 = 0$$

$$\therefore x^2 + \frac{2x}{k} + \frac{1}{k^2} - \frac{1}{k^2+1} = 0$$

$$\therefore \left(x + \frac{1}{k}\right)^2 - \left(\frac{1}{k^2} - 1\right) = 0$$

$$\therefore \quad \left(x + \frac{1}{k}\right)^2 = \frac{1-k^2}{k^2}$$

$$\therefore \quad x + \frac{1}{k} = \pm\frac{1}{k}\sqrt{1-k^2}$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1-k^2}}{k}$$

i $x^2 - 3kx + 1 = 0$

$$\therefore x^2 - 3kx + \frac{9}{4}k^2 - \left(\frac{9}{4}k^2 - 1\right) = 0$$

$$\therefore \quad \left(x - \frac{3k}{2}\right)^2 - \left(\frac{9}{4}k^2 - 1\right) = 0$$

$$\therefore \quad \left(x - \frac{3k}{2}\right)^2 = \left(\frac{9}{4}k^2 - 1\right)$$

$$\therefore \quad x - \frac{3k}{2} = \pm\sqrt{\frac{9}{4}k^2 - 1}$$

$$\therefore \quad x = \frac{3k \pm \sqrt{9k^2 - 4}}{2}$$

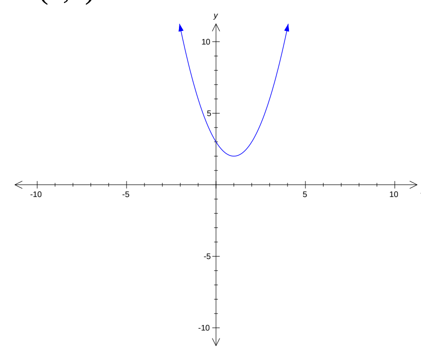
4

a $x^2 - 2x + 3$

$= x^2 - 2x + 1 + 2$

$= (x - 1)^2 + 2$

TP at (1,2)

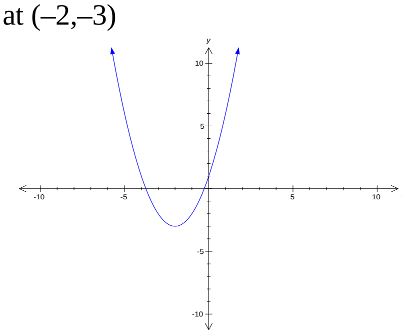


b $x^2 + 4x + 1$

$= x^2 + 4x + 4 - 3$

$= (x + 2)^2 - 3$

TP at (–2,–3)

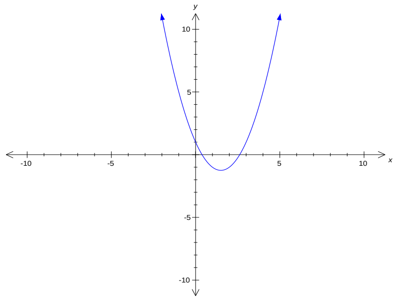


c $x^2 - 3x + 1$

$= x^2 - 3x + \frac{9}{4} - \frac{5}{4}$

$= \left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$

TP at $\left(\frac{3}{2}, -\frac{5}{4}\right)$

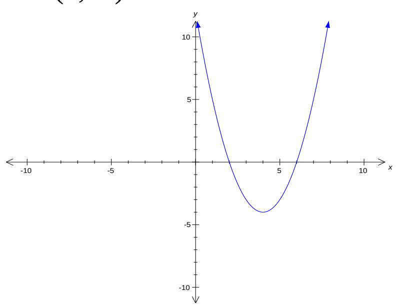


d $x^2 - 8x + 12$

$= x^2 - 8x + 16 - 4$

$= (x - 4)^2 - 4$

TP at (4,–4)

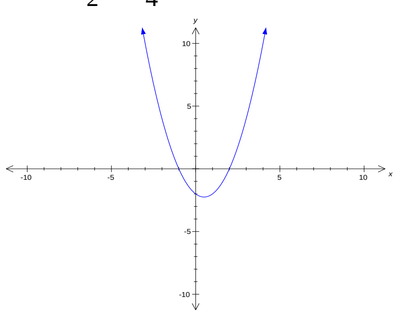


e $x^2 - x - 2$

$= x^2 - x + \frac{1}{4} - \frac{9}{4}$

$= \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$

TP at $(\frac{1}{2}, -\frac{9}{4})$



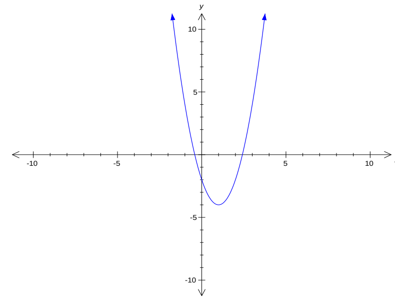
f $2x^2 + 4x - 2$

$= 2(x^2 + 2x - 1)$

$= 2(x^2 + 2x + 1 - 2)$

$= 2(x + 1)^2 - 4$

TP at (–1,–4)



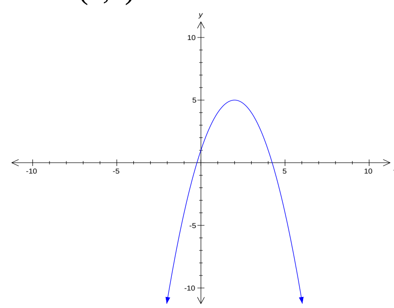
g $-x^2 + 4x + 1$

$= -(x^2 - 4x - 1)$

$= -(x^2 - 4x + 4 - 5)$

$= -(x - 2)^2 + 5$

TP at (2,5)



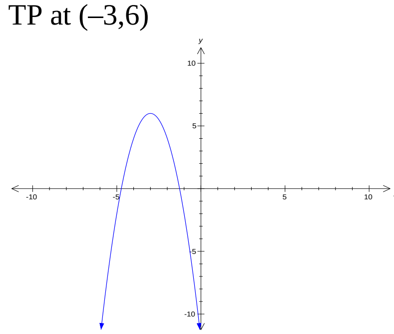
h $-2x^2 - 12x - 12$

$= -2(x^2 + 6x + 6)$

$= -2(x^2 + 6x + 9 - 3)$

$= -2(x + 3)^2 + 6$

TP at (–3,6)



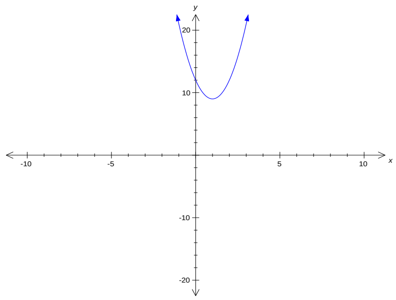
i $3x^2 - 6x + 12$

$= 3(x^2 - 2x + 4)$

$= 3(x^2 - 2x + 1 + 3)$

$= 3(x - 1)^2 + 9$

TP at (1,9)


y
20
10
-10
-5
5
10
x
-10
-20

Exercise 4F Solutions

1

a x-axis intercepts 4 and 10;

x-coordinate of vertex = $\frac{1}{2}(4 + 10) = 7$

b x-axis intercepts 6 and 8;

x-coordinate of vertex = $\frac{1}{2}(6 + 8) = 7$

c x-axis intercepts –6 and 8;

x-coordinate of vertex = $\frac{1}{2}(-6 + 8) = 1$

2

a x-axis intercepts a and 6;

x-coordinate of vertex = $\frac{1}{2}(a + 6) = 2$

$\therefore a + 6 = 4, \therefore a = -2$

b x-axis intercepts a and –4;

x-coordinate of vertex = $\frac{1}{2}(a - 4) = 2$

$\therefore a - 4 = 4, \therefore a = 8$

c x-axis intercepts a and 0;

x-coordinate of vertex = $\frac{1}{2}(a + 0) = -2$

$\therefore a = -4$

3

a $y = x^2 - 1$

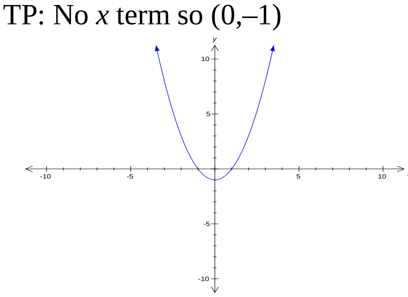
x-intercepts: $y = x^2 - 1 = 0$

$\therefore (x - 1)(x + 1) = 0$

$\therefore x = 1, -1$

x-int: (–1,0) and (1,0)

TP: No x term so (0,–1)



b $y = x^2 + 6x$

x-intercepts: $y = x^2 + 6x = 0$

$\therefore x(x + 6) = 0$

$\therefore x = 0, -6$

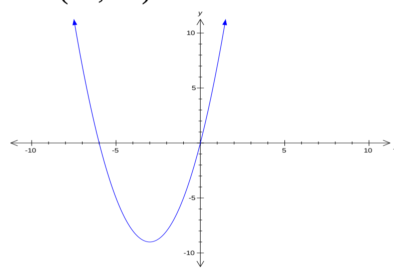
x-int: (–6,0) and (0,0)

TP: $y = x^2 + 6x$

$= x^2 + 6x + 9 - 9$

$= (x + 3)^2 - 9$

TP at (–3, –9)



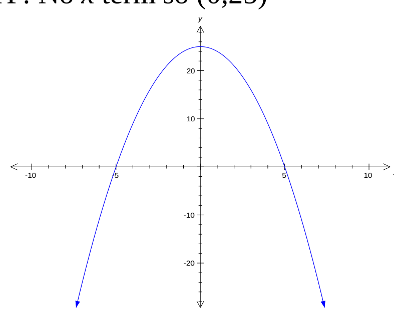
c $y = 25 - x^2$

x-intercepts: $y = 25 - x^2 = 0$

$\therefore (5 - x)(5 + x) = 0$

$\therefore x = 5, -5$

TP: No x term so (0,25)



d $y = x^2 - 4$

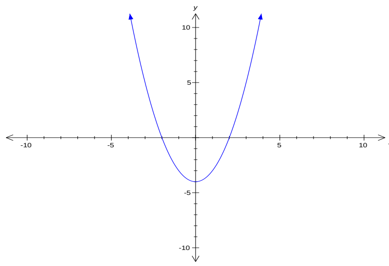
x-intercepts: $y = x^2 - 4 = 0$

$\therefore (x - 2)(x + 2) = 0$

$\therefore x = 2, -2$

x-int: (–2,0) and (2,0)

TP: No x term so (0,–4)



e $y = 2x^2 + 3x$

x-intercepts: $y = 2x^2 + 3x = 0$

$\therefore x(2x + 3) = 0$

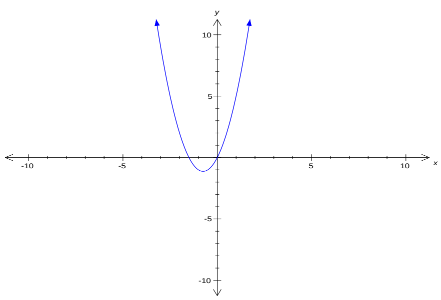
x-int: $(-\frac{3}{2}, 0)$ and (0,0)

TP: $y = 2x^2 + 3x$

$= 2\left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right)$

$= 2\left(x + \frac{3}{4}\right)^2 - \frac{9}{8}$

TP at $(-\frac{3}{4}, -\frac{9}{8})$



f $y = 2x^2 - 4x$

x-intercepts: $y = 2x^2 - 4x = 0$

$\therefore 2x(x - 2) = 0$

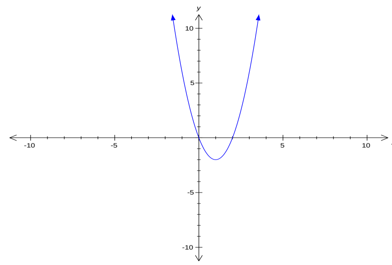
x-int: (2,0) and (0,0)

TP: $y = 2x^2 - 4x$

$= 2(x^2 - 2x + 1 - 1)$

$= 2(x - 1)^2 - 2$

TP at (1,–2)



g $y = -2x^2 - 3x$

x-intercepts: $y = -2x^2 - 3x = 0$

$\therefore -x(2x + 3) = 0$

x-int: $(-\frac{3}{2}, 0)$ and (0,0)

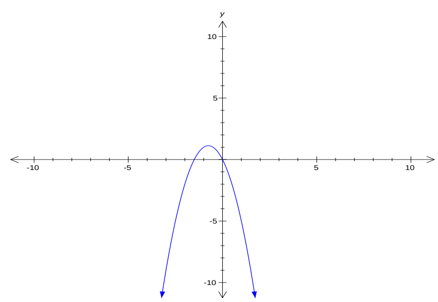
TP: $y = -2x^2 - 3x$

$= -2\left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right)$

$= -2\left(x + \frac{3}{4}\right)^2 + \frac{9}{8}$

$= -2\left(x + \frac{3}{4}\right)^2 + \frac{9}{8}$

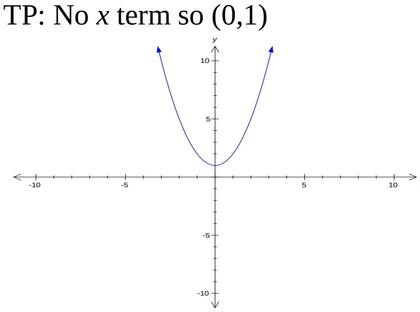
TP at $(-\frac{3}{4}, \frac{9}{8})$



h $y = x^2 + 1$

No x-intercepts since $y > 0$ for all x

TP: No x term so (0,1)



4

a $y = x^2 + 3x - 10$

x-intercepts: $y = x^2 + 3x - 10 = 0$

$\therefore (x + 5)(x - 2) = 0$

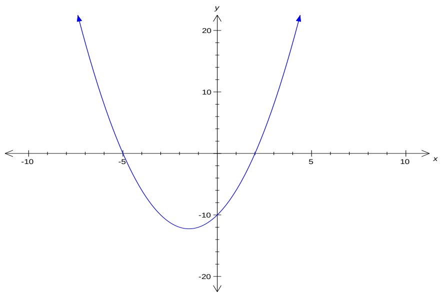
x-int: (–5, 0) and (2,0)

TP: $y = x^2 + 3x - 10$

$y = x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 10$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

TP at $\left(-\frac{3}{2}, -\frac{49}{4}\right)$



b $y = x^2 - 5x + 4$

x-intercepts: $y = x^2 - 5x + 4 = 0$

$\therefore (x - 1)(x - 4) = 0$

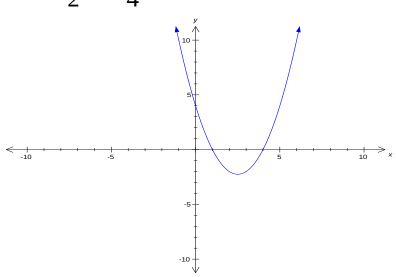
x-int: (1, 0) and (4,0)

TP: $y = x^2 - 5x + 4$

$y = x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 4$

$y = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$

TP at $\left(\frac{5}{2}, -\frac{9}{4}\right)$



c $y = x^2 + 2x - 3$

x-intercepts: $y = x^2 + 2x - 3$

$\therefore (x - 1)(x + 3) = 0$

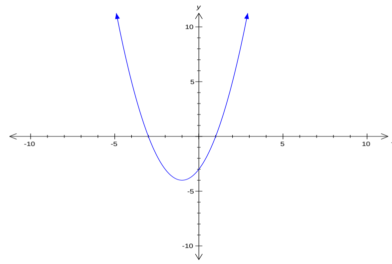
x-int: (1, 0) and (–3,0)

TP: $y = x^2 + 2x - 3$

$y = x^2 + 2x + 1 - 4$

$y = (x + 1)^2 - 4$

TP at (–1,–4)



d $y = x^2 + 4x + 3$

x-intercepts: $y = x^2 + 4x + 3 = 0$

$\therefore (x + 1)(x + 3) = 0$

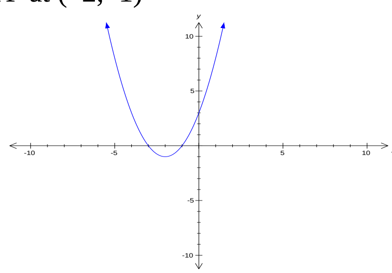
x-int: (–1, 0) and (–3,0)

TP: $y = x^2 + 4x + 3$

$y = x^2 + 4x + 4 - 1$

$y = (x + 2)^2 - 1$

TP at (–2,–1)



e $y = 2x^2 - x - 1$

x-intercepts: $y = 2x^2 - x - 1 = 0$

$\therefore (2x + 1)(x - 1) = 0$

x-int: $(-\frac{1}{2}, 0)$ and $(1,0)$

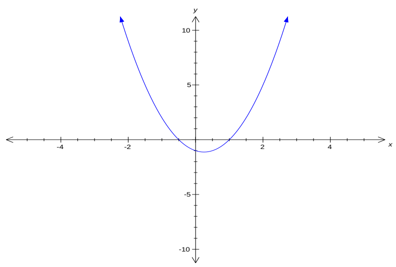
TP: $y = 2x^2 - x - 1$

$$y = 2\left(x^2 - \frac{x}{2} - \frac{1}{2}\right)$$

$$y = 2\left(x^2 - \frac{x}{2} + \frac{1}{16} - \frac{9}{16}\right)$$

$$y = 2\left(x - \frac{1}{4}\right)^2 - \frac{9}{8}$$

TP at $(\frac{1}{4}, -\frac{9}{8})$



f $y = 6 - x - x^2$

x-intercepts: $y = -(x^2 + x - 6) = 0$

$\therefore -(x + 3)(x - 2) = 0$

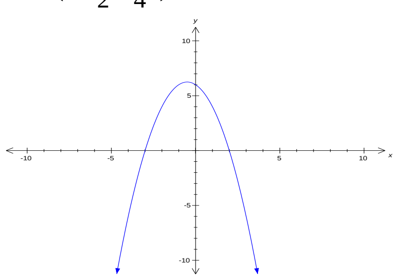
x-int: (–3, 0) and (2,0)

TP: $y = -(x^2 + x - 6)$

$$y = -\left(x^2 + x + \frac{1}{4} - 6 - \frac{1}{4}\right)$$

$$y = -\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$$

TP at $(-\frac{1}{2}, \frac{25}{4})$



g $y = -x^2 - 5x - 6$

x-intercepts: $y = -(x^2 + 5x + 6) = 0$

$\therefore -(x + 3)(x + 2) = 0$

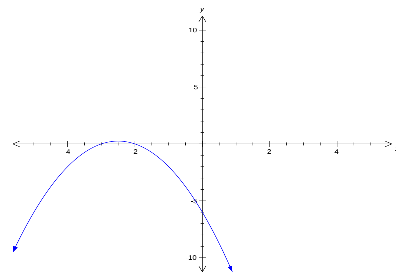
x-int: (–3, 0) and (–2,0)

TP: $y = -(x^2 + 5x + 6)$

$$y = -\left(x^2 + 5x + \frac{25}{4} + 6 - \frac{25}{4}\right)$$

$$y = -\left(x + \frac{5}{2}\right)^2 + \frac{1}{4}$$

TP at $(-\frac{5}{2}, \frac{1}{4})$



h $y = x^2 - 5x - 24$

x-intercepts: $y = x^2 - 5x - 24 = 0$

$\therefore (x + 3)(x - 8) = 0$

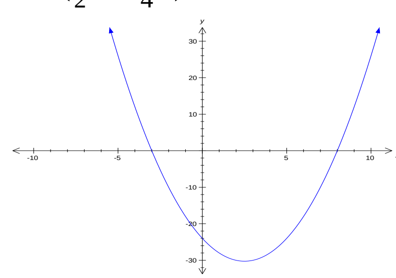
x-int: (–3, 0) and (8,0)

TP: $y = x^2 - 5x - 24$

$$y = x^2 - 5x + \frac{25}{4} - 24 - \frac{25}{4}$$

$$y = \left(x - \frac{5}{2}\right)^2 - \frac{121}{4}$$

TP at $(\frac{5}{2} - \frac{121}{4})$



Exercise 4G Solutions

1

a $a = 2$, $b = 4$ and $c = -3$

$$b^2 - 4ac = 4^2 - 4(-3)2 = 40$$

$$\sqrt{b^2 - 4ac} = \sqrt{40} = 2\sqrt{10}$$

b $a = 1$, $b = 10$ and $c = 18$

$$b^2 - 4ac = 10^2 - 4(18)1 = 28$$

$$\sqrt{b^2 - 4ac} = \sqrt{28} = 2\sqrt{7}$$

c $a = 1$, $b = 10$ and $c = -18$

$$b^2 - 4ac = 10^2 - 4(-18)1 = 172$$

$$\sqrt{b^2 - 4ac} = \sqrt{172} = 2\sqrt{43}$$

d $a = -1$, $b = 6$ and $c = 15$

$$b^2 - 4ac = 6^2 - 4(15)(-1) = 96$$

$$\sqrt{b^2 - 4ac} = \sqrt{96} = 4\sqrt{6}$$

e $a = 1$, $b = 9$ and $c = -27$

$$b^2 - 4ac = 9^2 - 4(-27)1 = 189$$

$$\sqrt{b^2 - 4ac} = \sqrt{189} = 3\sqrt{21}$$

2

a $$\frac{2 + 2\sqrt{5}}{2} = 1 + \sqrt{5}$$

b $$\frac{9 - 3\sqrt{5}}{6} = \frac{3 - \sqrt{5}}{2}$$

c $$\frac{5 + 5\sqrt{5}}{10} = \frac{1 + \sqrt{5}}{2}$$

d $$\frac{6 + 12\sqrt{2}}{6} = 1 + 2\sqrt{2}$$

3

a $x^2 + 6x = 4$

$$\therefore x^2 + 6x - 4 = 0$$

$$\therefore x = \frac{-6 \pm \sqrt{6^2 - 4(-4)1}}{2}$$

$$\therefore x = \frac{-6 \pm \sqrt{52}}{2}$$

$$\therefore x = -3 \pm \sqrt{13}$$

b $x^2 - 7x - 3 = 0$

$$\therefore x = \frac{7 \pm \sqrt{7^2 - 4(-3)1}}{2}$$

$$\therefore x = \frac{7 \pm \sqrt{61}}{2}$$

c $2x^2 - 5x + 2 = 0$

$$\therefore x = \frac{5 \pm \sqrt{5^2 - 4(2)2}}{4}$$

$$\therefore x = \frac{5 \pm \sqrt{9}}{4}$$

$$\therefore x = \frac{5 \pm 3}{4} = \frac{1}{2}, 2$$

d $2x^2 + 4x - 7 = 0$

$$\therefore x = \frac{-4 \pm \sqrt{4^2 - 4(-7)2}}{4}$$

$$\therefore x = \frac{-4 \pm \sqrt{72}}{4}$$

$$\therefore x = -1 \pm \frac{6}{4}\sqrt{2}$$

$$\therefore x = -1 \pm \frac{3}{2}\sqrt{2}$$

e $2x^2 + 8x = 1$

$$\therefore 2x^2 + 8x - 1 = 0$$

$$\therefore x = \frac{-8 \pm \sqrt{8^2 - 4(-1)2}}{4}$$

$$\therefore x = -2 \pm \frac{\sqrt{72}}{4}$$

$$\therefore x = -2 \pm \frac{3}{2}\sqrt{2}$$

f $5x^2 - 10x = 1$

$$\therefore 5x^2 - 10x - 1 = 0$$

$$\therefore x = \frac{10 \pm \sqrt{10^2 - 4(-1)5}}{10}$$

$$\therefore x = 1 \pm \frac{\sqrt{120}}{10}$$

$$\therefore x = 1 \pm \frac{\sqrt{30}}{5}$$

g $-2x^2 + 4x - 1 = 0$

$\therefore x = \dfrac{-4 \pm \sqrt{4^2 - 4(-1)(-2)}}{-4}$

$\therefore x = 1 \pm \dfrac{\sqrt{8}}{4}$

$\therefore x = 1 \pm \dfrac{\sqrt{2}}{2}$

h $2x^2 + x = 3$

$\therefore 2x^2 + x - 3 = 0$

$\therefore x = \dfrac{-1 \pm \sqrt{1^2 - 4(-3)2}}{4}$

$\therefore x = \dfrac{-1 \pm \sqrt{25}}{4}$

$\therefore x = \dfrac{-1 \pm 5}{4} \therefore x = 1, -\dfrac{3}{2}$

i $2.5x^2 + 3x + 0.3 = 0$

$\therefore x = \dfrac{-3 \pm \sqrt{3^2 - 4(0.3)2.5}}{5}$

$\therefore x = \dfrac{-3 \pm \sqrt{6}}{5}$

j $-0.6x^2 - 1.3x = 0.1$

$\therefore -6x^2 - 13x - 1 = 0$

$\therefore 6x^2 + 13x + 1 = 0$

$\therefore x = \dfrac{-13 \pm \sqrt{13^2 - 4(1)6}}{12}$

$\therefore x = \dfrac{-13 \pm \sqrt{145}}{12}$

k $2kx^2 - 4x + k = 0$

$\therefore x = \dfrac{4 \pm \sqrt{4^2 - 4(2k)k}}{4k}$

$\therefore x = 1 \pm \dfrac{\sqrt{16 - 8k^2}}{4k}$

$\therefore x = \dfrac{2 \pm \sqrt{4 - 2k^2}}{2k}$

l $2(1 - k)x^2 - 4kx + k = 0$

$\therefore x = \dfrac{4k \pm \sqrt{16k^2 - 8k(1 - k)}}{4(1 - k)}$

$\therefore x = \dfrac{4k \pm \sqrt{24k^2 - 8k}}{4(1 - k)}$

$\therefore x = \dfrac{2k \pm \sqrt{6k^2 - 2k}}{2(1 - k)}$

4 $S = 9.42r^2 + 6(6.28)r = 125.6$

$\therefore 9.42r^2 + 37.68r - 125.6 = 0$

$\therefore r = \dfrac{-37.68 \pm \sqrt{37.68^2 + 4(125.6)9.42}}{2(9.42)}$

$\therefore r = -2 \pm \dfrac{\sqrt{6152.4}}{18.84}$

Since r > 0,

$r = -2 + \dfrac{\sqrt{6152.4}}{18.84} = 2.16$ m

5

a $y = x^2 + 5x - 1$

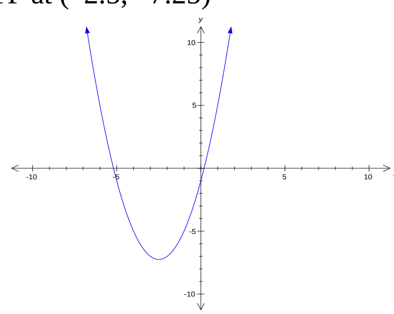
x-axis intercepts:

$x = \dfrac{-5 \pm \sqrt{29}}{2}$

$x = -\dfrac{5}{2};$

$y = \dfrac{25}{4} - \dfrac{25}{2} - 1 = -\dfrac{29}{4}$

TP at (–2.5, –7.25)



b $y = 2x^2 - 3x - 1$

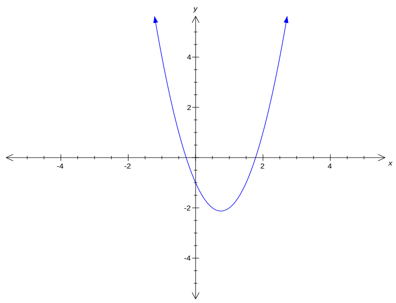
x-axis intercepts:

$\therefore x = \dfrac{3 \pm \sqrt{17}}{4}$

$x = \dfrac{3}{4}$;

$y = \dfrac{9}{8} - \dfrac{9}{4} - 1 = -\dfrac{17}{8}$

TP at (0.75, –2.125)



c $y = -x^2 - 3x + 1$

x-axis intercepts:

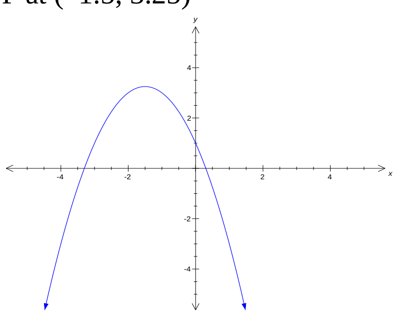
$\therefore x = \dfrac{3 \pm \sqrt{13}}{-2}$

$\therefore x = \dfrac{-3 \pm \sqrt{13}}{2}$

$x = -\dfrac{3}{2}$;

$y = -\dfrac{9}{4} + \dfrac{9}{2} + 1 = \dfrac{13}{4}$

TP at (–1.5, 3.25)



d $y + 4 = x^2 + 2x$

$\therefore y = x^2 + 2x - 4$

x-axis intercepts:

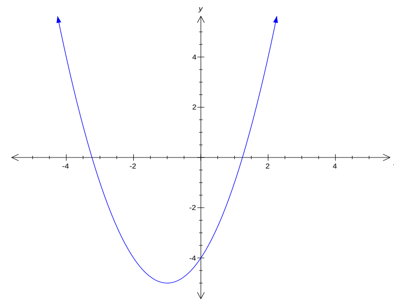
$\therefore x = \dfrac{-2 \pm \sqrt{20}}{2}$

$\therefore x = -1 \pm \sqrt{5}$

$x = -1$;

$y = 1 - 2 - 4 = -5$

TP at (–1, –5)



e $y = 4x^2 + 5x + 1$

x-axis intercepts:

$\therefore x = \dfrac{-5 \pm \sqrt{25 - 16}}{8}$

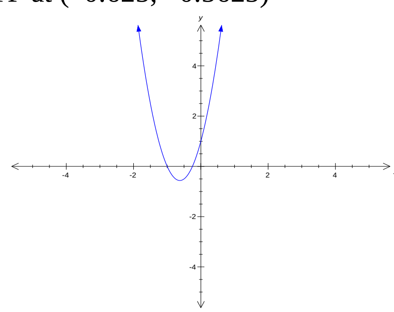
$\therefore x = \dfrac{-5 \pm 3}{8}$

$\therefore x = -1, -\dfrac{1}{4}$

$x = -\dfrac{5}{8}$;

$y = \dfrac{100}{64} - \dfrac{25}{8} + 1 = -\dfrac{9}{16}$

TP at (–0.625, –0.5625)



f $y = -3x^2 + 4x - 2$

x-axis intercepts:

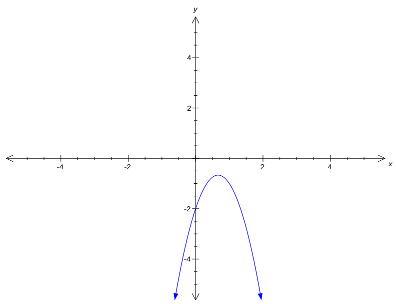
$\therefore\ x = \dfrac{-4 \pm \sqrt{16 - 24}}{-6}$

This is not defined, so no x-intercepts.

$x = \dfrac{2}{3}$;

$y = -\dfrac{4}{3} + \dfrac{8}{3} - 2 = -\dfrac{2}{3}$

TP at $(\dfrac{2}{3}, -\dfrac{2}{3})$



Exercise 4H Solutions

1

a $x^2 + 5x - 10 = 0$

Using $x_{n+1} = \dfrac{10}{x_n + 5}$ beginning at $x_0 = 1$

gives the sequence:

$1, \frac{5}{3}, \frac{3}{2}, \frac{20}{13} \ldots$ converging to 1.5311

b $x^2 - 3x - 5 = 0$

Using $x_{n+1} = \dfrac{5}{x_n - 3}$ beginning at $x_0 = -2$

gives the sequence:

$-2, -1, -\frac{5}{4}, -\frac{20}{17} \ldots$converging to -1.1926

c $x^2 + 2x - 7 = 0$

Using $x_{n+1} = \dfrac{7}{x_n + 2}$ beginning at $x_0 = 2$

gives the sequence:

$2, \frac{7}{4}, \frac{28}{15}, \frac{105}{58} \ldots$ converging to 1.8284

d $-x^2 - 2x + 5 = 0$

Using $x_{n+1} = \dfrac{5}{x_n + 2}$ beginning at $x_0 = 1$

gives the sequence:

$1, \frac{5}{3}, \frac{15}{11}, \frac{55}{37} \ldots$ converging to 1.4495

Exercise 4I Solutions

1

a $x^2 + 2x - 4;$

$\Delta = 2^2 - 4(-4) = 20$

b $x^2 + 2x + 4;$

$\Delta = 2^2 - 4(4) = -12$

c $x^2 + 3x - 4;$

$\Delta = 3^2 - 4(-4) = 25$

d $2x^2 + 3x - 4;$

$\Delta = 3^2 - 8(-4) = 41$

e $-2x^2 + 3x + 4;$

$\Delta = 3^2 - 4(-2)(4) = 41$

2

a $x^2 - 5x + 2;$

$\Delta = 5^2 - 4(2) = 17$

$\Delta > 0$ so graph crosses the x-axis

b $-4x^2 + 2x - 1;$

$\Delta = 2^2 - 4(-4)(-1) = -12$

$\Delta < 0$ so graph does not cross the x-axis

c $x^2 - 6x + 9;$

$\Delta = 6^2 - 4(9) = 10$

$\Delta = 0$ so graph touches the x-axis

d $-2x^2 - 3x + 8;$

$\Delta = 3^2 - 4(-2)\ 8 = 73$

$\Delta > 0$ so graph crosses the x-axis

e $3x^2 + 2x + 5;$

$\Delta = 2^2 - 4(5)(3) = -56$

$\Delta < 0$ so graph does not cross the x-axis

f $-x^2 - x - 1;$

$\Delta = 1^2 - 4(-1)(-1) = -3$

$\Delta < 0$ so graph does not cross the x axis

3

a $x^2 + 8x + 7;$

$\Delta = 8^2 - 4(7) = 36$

$\Delta > 0$ so the equation has 2 real roots

b $3x^2 + 8x + 7;$

$\Delta = 8^2 - 4(7)(3) = -20$

$\Delta < 0$ so the equation has 2 real roots

c $10x^2 - x - 3;$

$\Delta = 1^2 - 4(-3)(10) = 121$

$\Delta > 0$ so the equation has 2 real roots

d $2x^2 + 8x - 7;$

$\Delta = 8^2 - 4(-7)\ 2 = 120$

$\Delta > 0$ so the equation has 2 real roots

e $3x^2 - 8x - 7;$

$\Delta = 8^2 - 4(-7)\ 3 = 148$

$\Delta > 0$ so the equation has 2 real roots

f $10x^2 - x + 3;$

$\Delta = 1^2 - 4(10)(3) = -119$

$\Delta < 0$ so the equation has 2 real roots

4

a $9x^2 - 24x + 16;$

$\Delta = 24^2 - 4(9)(16) = 0$

$\Delta = 0$ so the equation has 1 rational root

b $-x^2 - 5x - 6;$

$\Delta = 5^2 - 4(-6)(-1) = 49$

$\Delta > 0$ so the equation has 2 rational roots, and is a perfect square

c $x^2 - x - 4;$

$\Delta = 1^2 - 4(-4) = 17$

$\Delta>0$ so the equation has 2 irrational roots, and is not a perfect square

d $25x^2 - 20x + 4;$

$\Delta = 20^2 - 4(25)(4) = 0$

$\Delta = 0$ so the equation has 1 rational root

e $6x^2 - 3x - 2$;

$\Delta = 3^2 - 4(6)(-2) = 57$

$\Delta > 0$ so the equation has 2 irrational roots and is not a perfect square

f $x^2 + 3x + 2$;

$\Delta = 3^2 - 4(2) = 1$

$\Delta > 0$ so the equation has 2 rational roots and is a perfect square

5 $4x^2 + (m - 4)x - m = 0$

$$\begin{aligned}\Delta &= (m-4)^2 - 4(4)(-m)\\ &= m^2 - 8m + 16 + 16m\\ &= m^2 + 8m + 16\\ &= (m+4)^2\end{aligned}$$

$\therefore \Delta \geq 0$ for all $m \in R$

So the equation always has rational solutions.

Exercise 4J Solutions

1

a $x^2 + 2x - 8 = 0$

$\therefore (x+2)(x-4) = 0$

$\therefore x = -2, 4$

Upright quadratic:

$\{x: x \le -2\} \cup \{x: x \ge 4\}$

b $x^2 - 5x - 24 = 0$

$\therefore (x+3)(x-8) = 0$

$\therefore x = -3, 8$

Upright quadratic:

$\{x: -3 < x < 8\}$

c $x^2 - 4x - 12 = 0$

$\therefore (x+2)(x-6) = 0$

$\therefore x = -2, 6$

Upright quadratic:

$\{x: -2 \le x \le 6\}$

d $2x^2 - 3x - 9 = 0$

$\therefore (2x+3)(x-3) = 0$

$\therefore x = -\frac{3}{2}, 3$

Upright quadratic:

$\{x: x < -2\} \cup \{x: x > 4\}$

e $6x^2 + 13x < -6$

$\therefore 6x^2 + 13x + 6 < 0$

$6x^2 + 13x + 6 = 0$

$\therefore (3x+2)(2x+3) = 0$

$\therefore x = -\frac{2}{3}, -\frac{3}{2}$

Upright quadratic:

$\{x: -\frac{3}{2} < x < -\frac{2}{3}\}$

f $-x^2 - 5x - 6 = 0$

$\therefore -(x+2)(x+3) = 0$

$\therefore x = -2, -3$

Inverted quadratic:

$\{x: -3 \le x \le -2\}$

g $12x^2 + x > 6$

$\therefore 12x^2 + x - 6 > 0$

$12x^2 + x - 6 = 0$

$\therefore (4x+3)(3x-2) = 0$

$\therefore x = -\frac{4}{3}, \frac{3}{2}$

Upright quadratic:

$\{x: x < -\frac{4}{3}\} \cup \{x: x > \frac{3}{2}\}$

h $10x^2 - 11x \le -3$

$\therefore 10x^2 - 11x + 3 \le 0$

$10x^2 - 11x + 3 = 0$

$\therefore (5x-3)(2x-1) = 0$

$\therefore x = \frac{1}{2}, \frac{3}{5}$

Upright quadratic:

$\{x: \frac{1}{2} \le x \le \frac{3}{5}\}$

i $x(x-1) \le 20$

$\therefore x^2 - x - 20 \le 0$

$x^2 - x - 20 = 0$

$\therefore (x-5)(x+4) = 0$

$\therefore x = -4, 5$

Upright quadratic:

$\{x: -4 \le x \le 5\}$

j $4 + 5p - p^2 = 0$

$\therefore p = \dfrac{-5 \pm \sqrt{41}}{-2}$

Inverted quadratic:

$\{p: \dfrac{5-\sqrt{41}}{2} \le p \le \dfrac{5+\sqrt{41}}{2}\}$

k $3 + 2y - y^2 = 0$

$\therefore (1+y)(3-y) = 0$

$\therefore y = -1, 3$

Inverted quadratic:

$\{y: y < -1\} \cup \{y: y > 3\}$

l $x^2 + 3x \geq -2$

$\therefore x^2 + 3x + 2 \geq 0$

$x^2 + 3x + 2 = 0$

$\therefore (x + 2)(x + 1) = 0$

$\therefore x = -2; -1$

Upright quadratic:

$\{x: x \leq -2\} \cup \{x: x \geq -1\}$

2

a $x^2 - 4mx + 20 = 0$

$\Delta = 16m^2 - 80 = 16(m^2 - 5)$

i If $(m^2 - 5) < 0$, no real solutions:
$\{m: -\sqrt{5} < m < \sqrt{5}\}$

ii If $(m^2 - 5) = 0$, one real solution:
$\{m: m = \pm\sqrt{5}\}$

iii If $(m^2 - 5) > 0$, 2 distinct solutions:
$\{m: m < -\sqrt{5}\} \cup \{m: m > \sqrt{5}\}$

b $mx^2 - 3mx + 3 = 0$

$\Delta = 9m^2 - 12m = 3m(3m - 4)$

i If $\Delta < 0$, no real solutions:

$\Delta = 0$ at $m = 0, \frac{4}{3}$

Upright parabola, so $\{m: 0 < m < \frac{4}{3}\}$

ii If $\Delta = 0$, one real solution;

$m = 0, \frac{4}{3}$ satisfies this, but there is no solution to the equation if $m = 0$, so $\{m: m = \frac{4}{3}\}$

iii If $(3m^2 - 4) > 0$, 2 distinct solutions:

$\{m: m < 0\} \cup \{m: m > \frac{4}{3}\}$

c $5x^2 - 5mx - m = 0$

$\Delta = 25m^2 + 20m = 5m(5m + 4)$

i If $5m(5m + 4) < 0$, no real solutions

$\Delta = 0$ at $m = 0, -\frac{4}{5}$

Quadratic in m is upright: $\{m: -\frac{4}{5} < m < 0\}$

ii If $5m(5m + 4) = 0$, one real solution:

$\{m: m = 0, -\frac{4}{5}\}$

iii If $5m(5m + 4) > 0$, 2 distinct solutions:

$\{m: m < -\frac{4}{5}\} \cup \{m: m > 0\}$

d $x^2 + 4mx - 4(m - 2) = 0$

$\Delta = 16m^2 + 16(m - 2)$

$= 16(m^2 + m - 2)$

i If $m^2 + m - 2 < 0$, no real solutions:
$m^2 + m - 2 = (m + 2)(m - 1)$
Quadratic in m is upright, so
$\{m: -2 < m < 1\}$

ii If $m^2 + m - 2 = 0$, one real solution:
$\{m: m = -2, 1\}$

iii If $m^2 + m - 2 > 0$, 2 distinct solutions:
$\{m: m < -2\} \cup \{m: m > 1\}$

3 $px^2 + 2(p + 2)x + p + 7 = 0$

$\Delta = 4(p + 2)^2 - 4p(p + 7)$

$= 4p^2 + 16p + 16 - 4p^2 - 28p$

$= 16 - 12p = 4(4 - 3p)$

This equation has no real solution if

$\Delta < 0$, i.e. if $p > \frac{4}{3}$

4 $(1 - 2p)x^2 + 8px - (2 + 8p) = 0$

$\Delta = 64p^2 + 4(1 - 2p)(2 + 8p)$

$= 64p^2 - 8(2p - 1)(4p + 1)$

$= 64p^2 - 8(8p^2 - 2p - 1)$

$= 8(2p + 1)$

This equation has one real solution if $\Delta = 0$;

$2p + 1 = 0$ or $p = -\frac{1}{2}$

5 $y = px^2 + 8x + p - 6$

$\Delta = 64 - 4p(p - 6)$

$= 4(-p^2 + 6p + 16)$

If the graph crosses the x-axis, $\Delta > 0$:

$\Delta = 0$ when $p = \frac{-6 \pm \sqrt{100}}{-2}$

$\therefore p = 3 \pm 5 = -2, 8$

Inverted quadratic, so $\Delta > 0$ when:

$\{p: -2 < p < 8\}$

6 $(p^2 + 1)x^2 + 2pqx + q^2 = 0$

$\Delta = 4p^2q^2 - 4q^2(p^2 + 1)$

$= 4q^2(p^2 - p^2 - 1)$

$= -4q^2$

This is negative for all values of *p*, and for all non–zero *q*, so there are no real solutions.

7 $mx^2 + (2m + n)x + 2n = 0$

$\Delta = (2m + n)^2 - 8mn$

$= 4m^2 + 4mn + n^2 - 8mn$

$= 4m^2 - 4mn + n^2$

$= (2m - n)^2$

This is a perfect square for all rational *m* and *n*, so the solution is rational also.

Exercise 4K Solutions

1

a $y = x^2 + 2x - 8$ and $y = 2 - x$

meet where $x^2 + 2x - 8 = 2 - x$

$\therefore x^2 + 3x - 10 = 0$

$\therefore (x + 5)(x - 2) = 0$

$\therefore x = -5, 2$

When $x = -5$, $y = 2 - (-5) = 7$

When $x = 2$, $y = 2 - 2 = 0$

Curves meet at (–5,7) and (2,0).

b $y = x^2 - x - 3$ and $y = 4x - 7$

meet where $x^2 - x - 3 = 4x - 7$

$\therefore x^2 - 5x + 4 = 0$

$\therefore (x - 4)(x - 1) = 0$

$\therefore x = 4, 1$

When $x = 1$, $y = 4 - 7 = -3$

When $x = 4$, $y = 16 - 7 = 9$

Curves meet at (1,–3) and (4,9).

c $y = x^2 + x - 5$ and $y = -x - 2$

meet where $x^2 + x - 5 = -x - 2$

$\therefore x^2 + 2x - 3 = 0$

$\therefore (x + 3)(x - 1) = 0$

$\therefore x = -3, 1$

When $x = -3$, $y = 3 - 2 = 1$

When $x = 1$, $y = -1 - 2 = -3$

Curves meet at (–3,1) and (1,–3).

d $y = x^2 + 6x + 6$ and $y = 2x + 3$

meet where $x^2 + 6x + 6 = 2x + 3$

$\therefore x^2 + 4x + 3 = 0$

$\therefore (x + 3)(x + 1) = 0$

$\therefore x = -3, -1$

When $x = -3$, $y = -6 + 3 = -3$

When $x = -1$, $y = -2 + 3 = 1$

Curves meet at (–3,–3) and (–1,1).

e $y = -x^2 - x + 6$ and $y = -2x - 2$

meet where $-x^2 - x + 6 = -2x - 2$

$\therefore -x^2 + x + 8 = 0$

$\therefore x^2 - x - 8 = 0$

$\therefore x = \dfrac{1 \pm \sqrt{1 - 4(8)}}{2}$

$\therefore x = \dfrac{1 \pm \sqrt{33}}{2}$

When $x = \dfrac{1 - \sqrt{33}}{2}$, $y = -3 + \sqrt{33}$

When $x = \dfrac{1 + \sqrt{33}}{2}$, $y = -3 - \sqrt{33}$

Curves meet at $\left(\dfrac{1 - \sqrt{33}}{2}, -3 + \sqrt{33}\right)$

and $\left(\dfrac{1 + \sqrt{33}}{2}, -3 - \sqrt{33}\right)$.

f $y = x^2 + x + 6$ and $y = 6x + 8$

meet where $x^2 + x + 6 = 6x + 8$

$\therefore x^2 - 5x - 2 = 0$

$\therefore x = \dfrac{5 \pm \sqrt{25 - 4(-2)}}{2}$

$\therefore x = \dfrac{5 \pm \sqrt{33}}{2}$

When $x = \dfrac{5 - \sqrt{33}}{2}$, $y = 23 - 3\sqrt{33}$

When $x = \dfrac{5 + \sqrt{33}}{2}$, $y = 23 + \sqrt{33}$.

Curves meet at $\left(\dfrac{5 - \sqrt{33}}{2}, 23 - 3\sqrt{33}\right)$

and $\left(\dfrac{5 + \sqrt{33}}{2}, 23 + 3\sqrt{33}\right)$.

2 If the straight line meets the parabola only once, then the $y_1 = y_2$ quadratic will produce a perfect square.

a $x^2 - 6x + 8 = -2x + 4$

$\therefore x^2 - 4x + 4 = 0$

$\therefore (x - 2)^2 = 0, \therefore x = 2$

Touches at (2,0).

b $x^2 - 2x + 6 = 4x - 3$

$\therefore x^2 - 6x + 9 = 0$

$\therefore (x - 3)^2 = 0, \therefore x = 3$

Touches at (3,9).

c $2x^2 + 11x + 10 = 3x + 2$

$\therefore 2x^2 + 8x + 8 = 0$

$\therefore 2(x + 2)^2 = 0, \therefore x = -2$

Touches at (–2,–4).

d $x^2 + 7x + 4 = -x - 12$

$\therefore x^2 + 8x + 16 = 0$

$\therefore (x + 4)^2 = 0, \therefore x = -4$

Touches at (–4,–8).

3

a $y = x^2 - 6x;\ y = 8 + x$

$\therefore x^2 - 6x = 8 + x$

$x^2 - 7x - 9 = 0$

$(x - 8)(x + 1) = 0$

$\therefore x = 8, -1$

$x = 8; y = 8 + 8 = 16$

$x = 1; y = 8 + 1 = 7$

b $y = 3x^2 + 9x;\ y = 32 - x$

$\therefore 3x^2 + 9x = 32 - x$

$3x^2 + 10x - 32 = 0$

$(3x + 16)(x - 2) = 0$

$\therefore x = -\frac{16}{3}, 2$

$x = -\frac{16}{3}; y = 32 + \frac{16}{3} = \frac{112}{3}$

$x = 2; y = 32 - 2 = 30$

c $y = 5x^2 + 9x;\ y = 12 - 2x$

$\therefore 5x^2 + 9x = 12 - 2x$

$5x^2 + 11x - 12 = 0$

$(5x - 4)(x + 3) = 0$

$\therefore x = \frac{4}{5}, -3$

$x = \frac{4}{5}; y = 12 - \frac{8}{5} = \frac{52}{5}$

$x = -3; y = 12 - (-6) = 18$

d $y = -3x^2 + 32x;\ y = 32 - 3x$

$\therefore -3x^2 + 32x = 32 - 3x$

$-3x^2 + 35x - 32 = 0$

$3x^2 - 35x + 32 = 0$

$(x - 1)(3x - 32) = 0$

$x = 1, \frac{32}{3}$

$x = 1; y = 32 - 3 = 29$

$x = \frac{32}{3}; y = 32 - 32 = 0$

e $y = 2x^2 - 12; y = 3(x - 4)$

$\therefore 2x^2 - 12 = 3x - 12$

$2x^2 - 3x = 0$

$x(2x - 3) = 0$

$x = 0, \frac{3}{2}$

$x = 0; y = 3(-4) = -12$

$x = \frac{3}{2}; y = 3\left(\frac{3}{2} - 4\right) = -\frac{15}{2}$

f $y = 11x^2; y = 21 - 6x$

$\therefore 11x^2 + 6x - 21 = 0$

$$\therefore x = \frac{-6 \pm \sqrt{6^2 - 4(-21)(11)}}{22}$$

$$= \frac{-3 \pm \sqrt{240}}{11} = \frac{-3 \pm 4\sqrt{15}}{11}$$

$x = \frac{-3 - 4\sqrt{15}}{11};$

$y = 21 + \frac{6}{11}(3 + 4\sqrt{15}) = \frac{249 + 24\sqrt{15}}{11}$

$x = \dfrac{-3 + 4\sqrt{15}}{11}$;

$y = 21 + \dfrac{6}{11}(3 - 4\sqrt{15}) = \dfrac{249 - 24\sqrt{15}}{11}$

Using a calculator: $x = 1.14, y = 14.19$;
$x = -1.68, y = 31.09$

4

a If $y = x + c$ is a tangent to the parabola $y = x^2 - x - 12$, then $x^2 - x - 12 = x + c$ must reduce to a quadratic with zero discriminant.

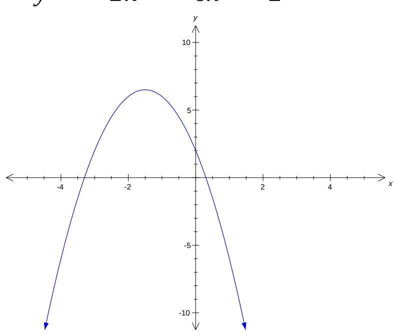
$x^2 - x - 12 = x + c$

$\therefore x^2 - 2x - (12 + c) = 0$

$\therefore \Delta = 4 + 4(12 + c)$

$= 4c + 52 = 0, \therefore c = -13$

b **i** $y = -2x^2 - 6x + 2$



ii If $y = mx + 6$ is a tangent to the parabola,

$-2x^2 - 6x + 2 = mx + 6$

$\therefore -2x^2 - (6 + m)x - 4 = 0$

$\therefore 2x^2 + (6 + m)x + 4 = 0$

For a tangent, $\Delta = 0$:

$\therefore \Delta = (6 + m)^2 - 4(4)(2) = 0$

$\therefore (6 + m)^2 = 32$

$\therefore 6 + m = \pm\sqrt{32} = \pm 4\sqrt{2}$

$\therefore m = -6 \pm 4\sqrt{2}$

5

a $y = x^2 + 3x$ has as a tangent $y = 2x + c$

$\Delta = 0$ for $x^2 + 3x = 2x + c$

$\therefore x^2 + x - c = 0$

$\therefore \Delta = 1 + 4c = 0, \therefore c = -\frac{1}{4}$

b For two intersections, $\Delta > 0$ so $c > -\frac{1}{4}$

6 $y = x$ is a tangent to the parabola

$y = x^2 + ax + 1$

$\therefore x^2 + ax + 1 = x$

$\therefore x^2 + (a - 1)x + 1 = 0$

$\Delta = (a - 1)^2 - 4 = 0$

$\therefore a - 1 = \pm 2$

$\therefore a = 1 \pm 2 = -1, 3$

7 $y = -x$ is a tangent to the parabola

$y = x^2 + x + b$

$\therefore x^2 + x + b = -x$

$\therefore x^2 + 2x + b = 0$

$\Delta = 4 - 4b = 0$

$\therefore b = 1$

8 A straight line passing through the point $(1, -2)$ has the form $y - (-2) = m(x - 1)$

$\therefore y = m(x - 1) - 2$

If this line is a tangent to $y = x^2$ then

$x^2 = m(x - 1) - 2$

$\therefore x^2 - m(x - 1) + 2 = 0$

$\therefore x^2 - mx + m + 2 = 0$

$\Delta = 0$ for a tangent here:

$\Delta = m^2 - 4(m + 2)$

$= m^2 - 4m - 8$

$m^2 - 4m - 8 = 0$

$\therefore m = \dfrac{4 \pm \sqrt{16 + 32}}{2}$

$\therefore m = 2 \pm \sqrt{12} = 2 \pm 2\sqrt{3}$

$\therefore y = (2 \pm 2\sqrt{3})(x - 1) - 2$

$y = 2(1 + \sqrt{3})x - 4 - 2\sqrt{3}$ and

$y = 2(1 - \sqrt{3})x - 4 + 2\sqrt{3}$

Exercise 4L Solutions

1 $y = ax^2$ passes through (2,8).

$\therefore 8 = a(2)^2, \therefore a = 2$

2 $y = ax^2 + c$ passes through (0,8) and (–1,4).

$\therefore a(0)^2 + c = 8, \therefore c = 8$

$a(-1)^2 + 8 = 4, \therefore a = -4$

3 $y = ax^2 + bx$ passes through (6,0) and (–1,4).

$\therefore a(6)^2 + 6b = 0$

$\therefore 36a + 6b = 0, \therefore b = -6a$

$a(-1)^2 - 6a(-1) = 4$

$\therefore 7a = 4$

$\therefore a = \frac{4}{7}; b = -\frac{24}{7}$

4 $y = a(x - b)^2 + c$

The vertex is at (1,6) so $y = a(x-1)^2 + 6$

$y = a(x-1)^2 + 6$ passes through (2,4)

$\therefore a(2 - 1)^2 + 6 = 4$

$\therefore a = -2; b = 1; c = 6$

5

a $y = a(x - b)^2 + c$

The vertex is at (0,5) so $y = (x-0)^2 + 5$

$y = ax^2 + 5$

$y = ax^2 + 5$ passes through (0,4)

$\therefore a(4)^2 + 5 = 0$

$\therefore a = -\frac{5}{16}$

$y = -\frac{5x^2}{16} + 5$

b $y = a(x - b)^2 + c$

The vertex is at (0,0) so $y = ax^2$

$y = ax^2$ passes through (–3,9)

$\therefore a(-3)^2 = 9$

$\therefore a = 1$

$y = x^2$

c $y = ax^2 + bx + c$

This is of the form $y = ax(x+7)$

For (4,4)

$4 = a(4)(4+7)$

$4 = 44a$

Therefore $a = \frac{1}{11}$

And the rule is $y = \frac{x^2}{11} + \frac{7}{11}$

d $y = a(x + b)(x + c)$

From x-intercepts, a and b are –1 and –3:

$y = a(x - 1)(x - 3)$

From y-intercept,

$a(-1)(-3) = 3, \therefore a = 1$

$\therefore y = (x - 1)(x - 3)$

e $y = a(x - b)^2 + c$

The vertex is at (–1, 5) so

$y = a(x+1)^2 + 5$

$y = a(x + 1)^2 + 5$ passes through (1, 0)

$\therefore a(2)^2 + 5 = 0$

$\therefore a = -\frac{5}{4}$

$y = -\frac{5}{4}(x + 1)^2 + 5$

OR $y = -\frac{5}{4}x^2 - \frac{5}{2}x + \frac{15}{4}$

Check with 3rd pt: $y = 0$ at $x = -3$

f $y = a(x - b)^2 + c$

The vertex is at (2, 2) so

$y = a(x+2)^2 + 2$

$y = a(x - 2)^2 + 2$ passes through (0,6)

$\therefore a(-2)^2 + 2 = 6$

$\therefore a = 1$

$y = (x - 2)^2 + 2$

OR $y = x^2 - 4x + 6$
Check with 3rd pt: $y = 6$ at $x = 4$

6 $y = a(x - b)^2 + c$
The vertex is at (–1, 3) so
$y = a(x+1)^2 + 3$
$y = a(x + 1)^2 + 3$ passes through (3,8)
$\therefore\ a(4)^2 + 3 = 8$
$\therefore\ 16a = 5,\ \therefore\ a = \frac{5}{16}$
$y = \frac{5}{16}(x + 1)^2 + 3$

7 $y = a(x + b)(x + c)$
From x intercepts, a and b are 6 and –3:
$y = a(x - 6)(x + 3)$
Using (1, 10):
$a(1 - 6)(1 + 3) = 10$
$\therefore\ -20a = 10,\ \therefore\ a = -\frac{1}{2}$
$\therefore\ y = -\frac{1}{2}(x - 6)(x + 3)$
OR $y = -\frac{1}{2}(x^2 - 3x - 18)$

8 $y = a(x - b)^2 + c$
The vertex is at (–1, 3) so
$y = a(x+1)^2 + 3$
$y = a(x + 1)^2 + 3$ passes through (0,4)
$\therefore\ a + 3 = 4,\ \therefore\ a = 1$
$y = (x + 1)^2 + 3$
OR $y = x^2 + 2x + 4$

9 The suspension cable forms a parabola:
$y = a(x - b)^2 + c$
The vertex is at (90, 30) so
$y = a(x - 90)^2 + 30$
When $x = 0$, $y = 75$, so:
$y = a(-90)^2 + 30 = 75$
$\therefore\ 8100a = 45,\ \therefore\ a = \frac{1}{180}$
$y = \frac{1}{180}(x - 90)^2 + 30$
$\therefore\ y = \frac{1}{180}x^2 - x + 75$

10

a $y = \frac{1}{3}(x + 4)(8 - x)$

Squared term is negative, so inverted parabola; must be **A** or **C**.
The x-intercepts must be at 8 and –4, so **C**.

b $y = x^2 - x + 2$

Positive squared term gives an upright parabola; must be **B** or **D**.
The y-intercept is at (0,2) so only **B** is possible

c $y = -10 + 2(x - 1)^2$

Positive squared term gives an upright parabola; must be **B** or **D**.
Vertex is at (1,–10) so **D**.

d $y = \frac{1}{2}(9 - x^2)$

Squared term is negative so inverted parabola; must be A or C.
Vertex at $(0, \frac{9}{2})$ so **A**..

11 $y = 2(x - b)^2 + c$

(1, −2) = TP (vertex) = (b,c)

$\therefore y = 2(x - 1)^2 - 2$

OR $y = 2x^2 - 4x$

12 $y = a(x - b)^2 + c$

(1, −2) = TP (vertex) = (b,c)

$\therefore y = a(x - 1)^2 - 2$

Using the point (3,2),

$a(3 - 1)^2 - 2 = 2$

$\therefore 4a - 2 = 2, \therefore a = 1$

$\therefore y = (x - 1)^2 - 2$

OR $y = x^2 - 2x - 1$

13 $y = a(x - b)^2 + c$

(2, 2) = TP (vertex) = (b,c)

$\therefore y = a(x - 2)^2 + 2$

Using the point (4,–6),

$a(4 - 2)^2 + 2 = -6$

$\therefore 4a + 2 = -6 \therefore, a = -2$

$\therefore y = -2(x - 2)^2 + 2$

OR $y = -2x^2 + 8x - 6$

14 **(a)** has x-intercepts at 0 and 10, so

$y = a(x - b)(x - c)$

$b = 0$, $c = 10$

$\therefore y = ax(x - 10)$

$a > 0$ because upright parabola

(b) has x-intercepts at –4 and 10, so

$y = a(x - b)(x - c)$

$b = -4$, $c = 10$

$\therefore y = a(x + 4)(x - 10)$

$a<0$ because upright parabola

(c) has no x intercepts, so

$y = a(x - b)^2 + c$

Vertex is at (6,6) so $b = c = 6$

$y = a(x - 6)^2 + 6$

y-intercept is at (0,8):

$a(0 - 6)^2 + 6 = 8$

$\therefore 36a = 2, \therefore a = \frac{1}{18}$

$y = \frac{1}{18}(x - 6)^2 + 6$

(d) $y = a(x - b)^2 + c$

Vertex is at (8,0) so $b = 8$; $c = 0$

$y = a(x - 8)^2$

$a < 0$ because inverted parabola

15 **(a)** $y = ax^2 + x + b$

Using D = (0,2), $b = 2$

Using A = (2,3),

$4a + 2 + 2 = 3$

$\therefore \quad a = -\frac{1}{4}$

$y = -\frac{1}{4}x^2 + x + 2$

(b) $y = ax^2 + x + b$

Using C = (0,–5), $b = -5$

Using B = (2,1),

$4a + 2 - 5 = 1$

$\therefore \quad a = 1$

$y = x^2 + x - 5$

16 $r = at^2 + bt + c$

(1) $t = 5$, $r = 3$: $25a + 5b + c = 3$
(2) $t = 9$, $r = 6$: $81a + 9b + c = 6$
(3) $t = 13$, $r = 5$: $169a + 13b + c = 5$
(2) – (1) gives $56a + 4b = 3$
(3) – (2) gives $88a + 4b = -1$
From these 2 equations, $32a = -4$

so $a = -\frac{1}{8}$

Substitute into $56a + 4b = 3$:

$$-\frac{56}{8} + 4b = 3$$

$$\therefore 4b = 10, \therefore b = \frac{5}{2}$$

Substitute into **(1)**:

$$-\frac{25}{8} + \frac{25}{2} + c = 3$$

$$\therefore c = -\frac{51}{8}$$

$$r = -\frac{1}{8}(t^2 - 20t + 51)$$

$$\therefore r = -\frac{1}{8}t^2 + \frac{5}{2}t - \frac{51}{8}$$

17

a $y = (x - 4)^2 - 3$
Upright parabola, vertex (4,–3) so **B**

b $y = 3 - (x - 4)^2$
Inverted parabola, vertex (4,3) so **D**

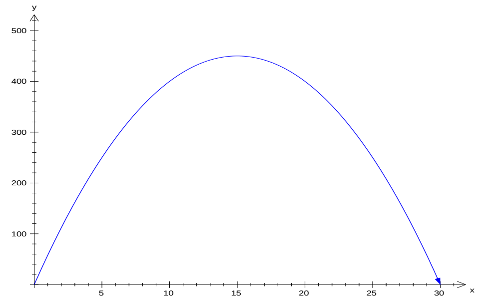
Exercise 4M Solutions

1

a Width of paddock = r; length = $60 - 2r$

$\therefore A = r(60 - 2r) = 60r - 2r^2$

b

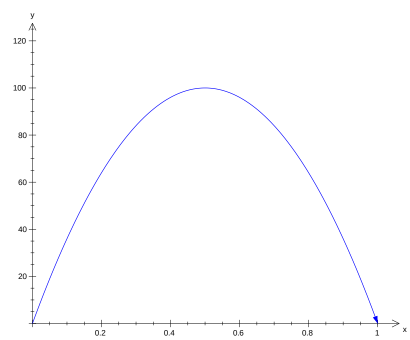


c Maximum area is at the vertex, i.e. when $r = 15$ (halfway between the two x-intercepts). When $r = 15$,

$A = 15(60 - 30) = 450\ \text{m}^2$

2 $E = 400(x - x^2)$

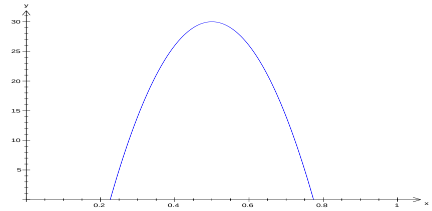
a



b Zero efficiency rating when $x = 0$ and 1

c Maximum efficiency rating is at the vertex where $x = 0.5$

d $E \geq 70$ when $400x - 400x^2 - 70 \geq 0$
i.e. $\{x: 0.23 < x < 0.77\}$



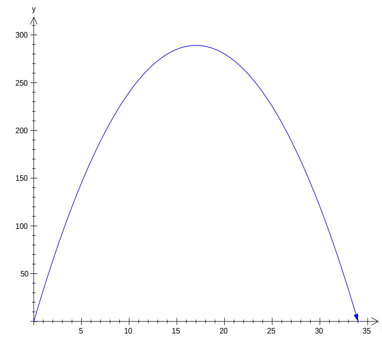
3

a If x cm = length of the rectangle, then

$2x + 2w = 68$, $\therefore w = 34 - x$

$A = lw = x(34 - x) = 34x - x^2$

b

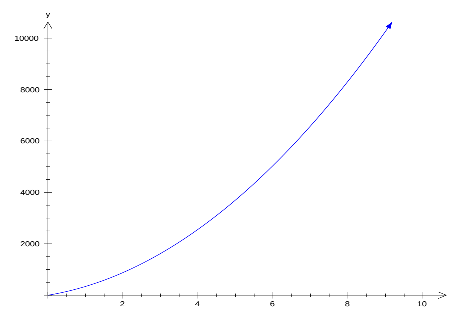


c Maximum area formed is at the vertex where $x = 17$:

$A = 17(34 - 17) = 172 = 289\ \text{cm}^2$

4 $C = 240h + 100h^2$

a



h is most unlikely to be less than zero in an alpine area, and will be less than 10, since the highest mountain on Earth is less high than 10 km above sea level.

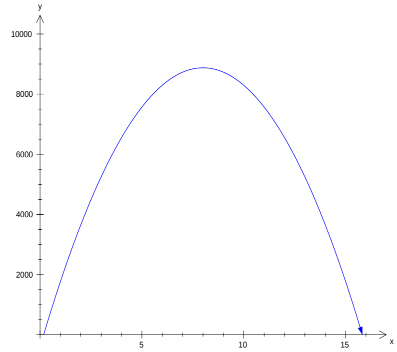
b C's maximum value is at the top of the highest peak in the mountains (8.848 km for Mt Everest).

c For $h = 2.5$ km,

$C = 240(2.5) + 100(2.5)^2$
$= 600 + 625 = \$1225$

5 $T = 290(8t - 0.5t^2 - 1.4)$

a

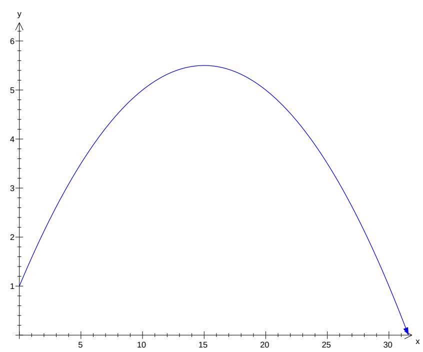


Solving $8t - 0.5t^2 - 1.4 = 0$ with a CAS gives t=0.18, 15.82. So $t \in (0.18, 15.82)$

b At the vertex t = 8, T = 8874 units

6

a $d = 1 + \frac{3}{5}x - \frac{1}{50}x^2$, $x \geq 0$



b **i** Maximum height = 5.5 m

ii When y = 2, $x = 15 \pm 5\sqrt{7}$
(x = 1.9 m or 28.1 m)

iii y-intercept = , so it was struck 1 metre above the ground.

7

a $y = ax^2 + bx + c$

$(-2, -1)$: $4a - 2b + c = -1$ …**(1)**

$(1, 2)$: $a + b + c = 2$ …**(2)**

$(3, -16)$ $9a + 3b + c = -16$ …**(3)**

(2) – (1) gives $3b - 3a = 3$ or $b = a + 1$

(3) – (2) gives $8a + 2b = -18$ or $b = -9 - 4a$

$b = a + 1 = -9 - 4a$

$\therefore 5a = -10$, $\therefore a = -2$; $b = -1$

Substitute into **(1):**

$-8 + 2 + c = -1 \therefore c = 5$

$y = -2x^2 - x + 5$

b $y = ax^2 + bx + c$

$(-1, -2)$: $a - b + c = -2$ …**(1)**

$(1, -4)$: $a + b + c = -4$ …**(2)**

$(3, 10)$: $9a + 3b + c = 10$ …**(3)**

(2) – (1) gives $2b = -2$ or $b = -1$

(3) – (2) gives $8a + 2b = 14$

$\therefore 8a = 16 \therefore a = 2$

Substitute into **(2):**

$2 - 1 + c = -4$, $\therefore c = -5$

$y = 2x^2 - x - 5$

c $y = ax^2 + bx + c$

$(-3, 5)$: $9a - 3b + c = 5$ …**(1)**

$(3, 20)$: $9a + 3b + c = 20$ …**(2)**

$(5, 57)$: $25a + 5b + c = 57$ …**(3)**

(2) – (1) gives $6b = 15$ or $b = \frac{5}{2}$

(3) – (2) gives $16a + 2b = 37$

$\therefore 16a + 5 = 37$, $\therefore a = 2$

Substitute into **(2):**

$18 + \frac{15}{2} + c = 20$, $\therefore c = -\frac{11}{2}$

$y = 2x^2 + \frac{5}{2}x - \frac{11}{2}$

8 The x-intercepts are 0 and 1.5

So $y = ax(x - 1.5)$

A is the point (0.75, 0.6) so:

$0.6 = a(0.75)(0.75 - 1.5)$

$\frac{3}{5} = -\frac{9}{16}a$

So $a = -16$

$y = -\frac{16}{15}x^2 + \frac{8}{5}x$

$a = -\frac{16}{15}$, $b = \frac{8}{5}$, $c = 0$

9

a $s = at^2 + bt + c$

$900a + 30b + c = 7.2$ …**(1)**

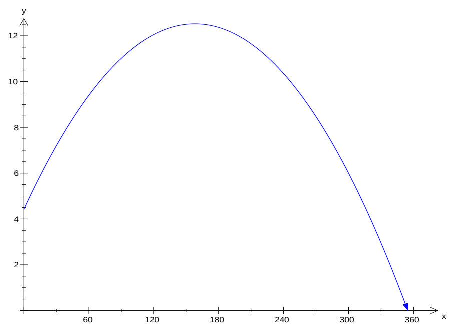
$22\,500a + 150b + c = 12.5$ …**(2)**

$90\,000a + 300b + c = 6$ …**(3)**

(2) – (1) gives $21600a + 120b = 5.3$

(3) – (2) gives $67500a + 150b = -6.5$
Using a CAS, the solution is:
$a = -\dfrac{7}{21\ 600}$; $b = \dfrac{41}{400}$; $c = \dfrac{53}{12}$

b



c **i** $t = 180$, $s = 12.3\dot{6}$, so spending is estimated at $1 236 666.
ii $t = 350$, $s = 0.59259$, so spending is estimated at $59259

Chapter Review: Multiple-choice Solutions

1 $12x^2 + 7x - 12 = (3x + 4)(4x - 3)$ **A**

2 $x^2 - 5x - 14 = 0$

$\therefore (x - 7)(x + 2) = 0$

$\therefore x = -2, 7$ **C**

3 $y = 8 + 2x - x^2$

$= 9 - (x^2 - 2x + 1)$

$= 9 - (x - 1)^2$

Maximum value of y is 9
when $x = 1$ **C**

4 $y = 2x^2 - kx + 3$

If the graph of touches the x–axis then $\Delta = 0$:

$\Delta = (-k)^2 - 24 = 0$

$\therefore k^2 = 24$

$\therefore k = \pm\sqrt{24} = \pm 2\sqrt{6}$ **E**

5 $x^2 - 56 = x$

$\therefore x^2 - x - 56 = 0$

$\therefore (x - 8)(x + 7) = 0$

$\therefore x = -7, 8$ **B**

6 $x^2 + 3x - 10$

$\Delta = 3^2 + 40 = 49$ **C**

7 $y = 3x^2 + 6x - 1$

$= 3x^2 + 6x + 3 - 4$

$= 3(x + 1)^2 - 4$

TP is at (–1, –4). **E**

8 $5x^2 - 10x - 2$

$= 5(x^2 - 2x + 1) - 7$

$= 5(x - 1)^2 - 7$ **E**

9 If two real roots of $mx^2 + 6x - 3 = 0$ exist, then $\Delta > 0$:

$\Delta = 6^2 + 12m = 12(m + 3)$

$m > -3$ **D**

10 $6x^2 - 8xy - 8y^2$

$= (3x + 2y)(2x - 4y)$ **A**

Chapter Review: Short-answer Solutions (technology-free)

1

a $x^2 + 9x + \frac{81}{4} = \left(x + \frac{9}{2}\right)^2$

b $x^2 + 18x + 81 = (x + 9)^2$

c $x^2 - \frac{4}{5}x + \frac{4}{25} = \left(x - \frac{2}{5}\right)^2$

d $x^2 + 2bx + b^2 = (x + b)^2$

e $9x^2 - 6x + 1 = (3x - 1)^2$

f $25x^2 + 20x + 4 = (5x + 2)^2$

2

a $-3(x - 2) = -3x + 6$

b $-a(x - a) = -ax + a^2$

c $(7a - b)(7a + b) = 49a^2 - b^2$

d $(x + 3)(x - 4) = x^2 + 3x - 4x - 12$

$= x^2 - x - 12$

e $(2x + 3)(x - 4) = 2x^2 + 3x - 8x - 12$

$= 2x^2 - 5x - 12$

f $(x + y)(x - y) = x^2 - y^2$

g $(a - b)(a^2 + ab + b^2)$

$= a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3$

$= a^3 - b^3$

h $(2x + 2y)(3x + y) = 6x^2 + 6xy + 2xy + 2y^2$

$= 6x^2 + 8xy + 2y^2$

i $(3a + 1)(a - 2) = 3a^2 + a - 6a - 2$

$= 3a^2 - 5a - 2$

j $(x + y)^2 - (x - y)^2$

$= ((x + y) - (x - y))((x + y) + (x - y))$

$= (2y)(2x) = 4xy$

k $u(v + 2) + 2v(1 - u)$

$= uv + 2u + 2v - 2uv$

$= 2u + 2v - uv$

l $(3x + 2)(x - 4) + (4 - x)(6x - 1)$

$= (3x + 2)(x - 4) + (x - 4)(1 - 6x)$

$= (x - 4)(3x + 2 + 1 - 6x)$

$= (x - 4)(3 - 3x)$

$= -3x^2 + 15x - 12$

3

a $4x - 8 = 4(x - 2)$

b $3x^2 + 8x = x(3x + 8)$

c $24ax - 3x = 3x(8a - 1)$

d $4 - x^2 = (2 - x)(2 + x)$

e $au + 2av + 3aw = a(u + 2v + 3w)$

f $4a^2b^2 - 9a^4 = a^2(4b^2 - 9a^2)$

$= a^2(2b - 3a)(2b + 3a)$

g $1 - 36x^2a^2 = (1 - 6ax)(1 + 6ax)$

h $x^2 + x - 12 = (x + 4)(x - 3)$

i $x^2 + x - 2 = (x + 2)(x - 1)$

j $2x^2 + 3x - 2 = (2x - 1)(x + 2)$

k $6x^2 + 7x + 2 = (3x + 2)(2x + 1)$

l $3x^2 - 8x - 3 = (3x + 1)(x - 3)$

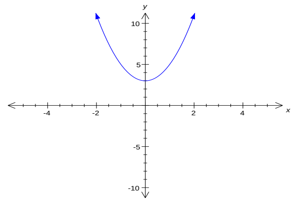
m $3x^2 + x - 2 = (3x - 2)(x + 1)$

n $6a^2 - a - 2 = (3a - 2)(2a + 1)$

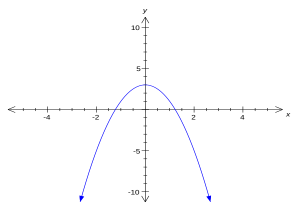
o $6x^2 - 7x + 2 = (3x - 2)(2x - 1)$

4

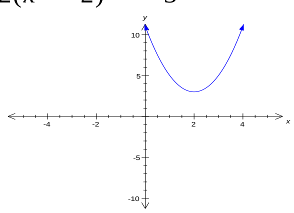
a $y = 2x^2 + 3$



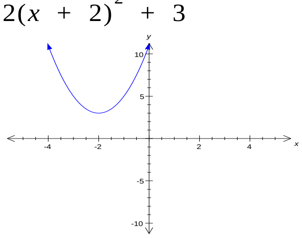
b $y = -2x^2 + 3$



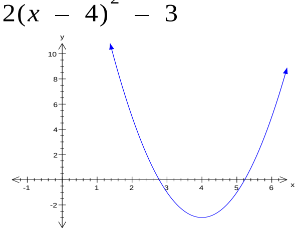
c $y = 2(x - 2)^2 + 3$



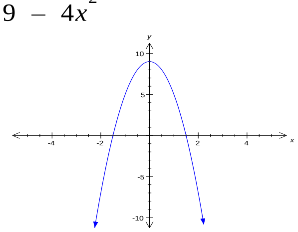
d $y = 2(x + 2)^2 + 3$



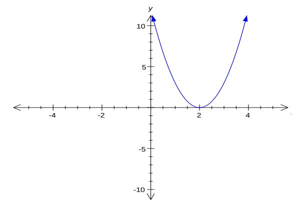
e $y = 2(x - 4)^2 - 3$



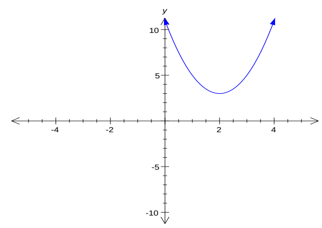
f $y = 9 - 4x^2$



g $y = 3(x - 2)^2$



h $y = 2(2 - x)^2 + 3$

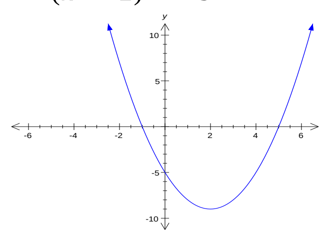


5

a $y = x^2 - 4x - 5$

$= x^2 - 4x + 4 - 9$

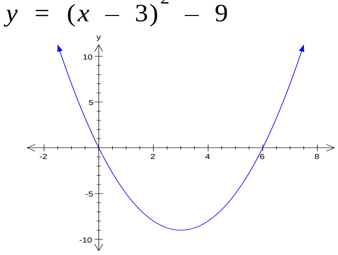
$\therefore y = (x - 2)^2 - 9$



b $y = x^2 - 6x$

$= x^2 - 6x + 9 - 9$

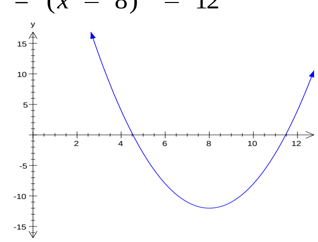
$\therefore y = (x - 3)^2 - 9$



c $y = x^2 - 8x + 4$

$= x^2 - 8x + 16 - 12$

$\therefore y = (x - 8)^2 - 12$

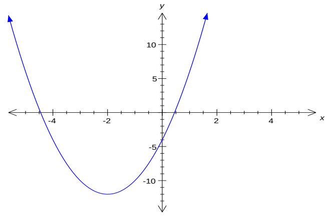


d $y = 2x^2 + 8x - 4$

$= 2(x^2 + 4x - 2)$

$\therefore y = 2(x^2 + 4x + 4 - 6)$

$\therefore y = 2(x + 2)^2 - 12$

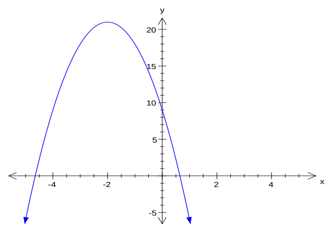


e $y = -3x^2 - 12x + 9$

$= -3(x^2 + 4x - 3)$

$= -3(x^2 + 4x + 4 - 7)$

$\therefore y = -3(x + 2)^2 + 21$

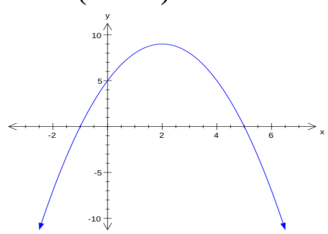


f $y = -x^2 + 4x + 5$

$\therefore y = -(x^2 - 4x - 5)$

$\therefore y = -(x^2 - 4x + 4 - 9)$

$\therefore y = -(x - 2)^2 + 9$



6 **i** y–intercepts are at $(0,c)$ in each case; x– intercepts are where the factors equal zero.

ii The axis of the symmetry is at

$x = -\dfrac{b}{2a}$

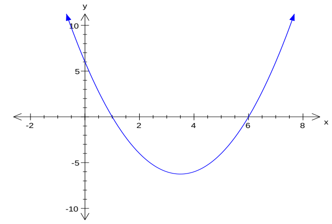
iii The turning point is on the axis of symmetry with the y-value for that point.

a $y = x^2 - 7x + 6 = (x - 6)(x - 1)$

i (0,6), (6,0) and (1,0)

ii $x = -\dfrac{b}{2a} = \dfrac{7}{2}$

iii Turning point at $(\dfrac{7}{2}, -\dfrac{25}{4})$



b $y = -x^2 - x + 12$

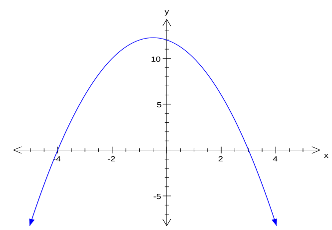
$= -(x^2 + x - 12)$

$= -(x + 4)(x - 3)$

i (0,12), (–4,0) and (3, 0)

ii $x = -\dfrac{b}{2a} = -\dfrac{1}{2}$

iii Turning point at $(-\dfrac{1}{2}, \dfrac{49}{4})$



c $y = -x^2 + 5x + 14$

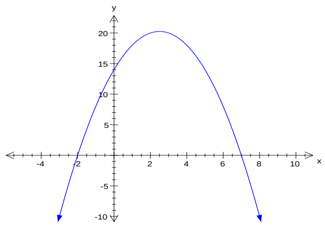
$= -(x^2 - 5x - 14)$

$= -(x - 7)(x + 2)$

i (0, 14), (–2, 0) and (7, 0)

ii $x = -\frac{b}{2a} = \frac{5}{2}$

iii turning point at $(\frac{5}{2}, \frac{81}{4})$

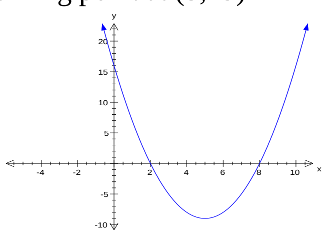


d $y = x^2 - 10x + 16 = (x - 8)(x - 2)$

i (0,16), (2,0) and (8,0)

ii $x = -\frac{b}{2a} = 5$

iii Turning point at (5,–9)

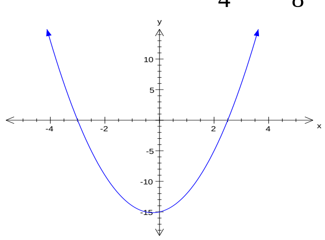


e $y = 2x^2 + x - 15 = (2x - 5)(x + 3)$

i (0,–15), $(\frac{5}{2}$,0) and (–3,0)

ii $x = -\frac{b}{2a} = -\frac{1}{4}$

iii Turning point at $(-\frac{1}{4}, -\frac{121}{8})$

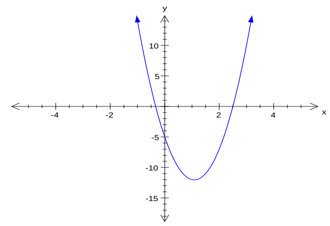


f $y = 6x^2 - 13x - 5 = (3x + 1)(2x - 5)$

i (0,–5), $(\frac{5}{2}$,0) and $(-\frac{1}{3}$,0)

ii $x = -\frac{b}{2a} = \frac{13}{12}$

iii Turning point at $(\frac{13}{12}, -\frac{289}{24})$

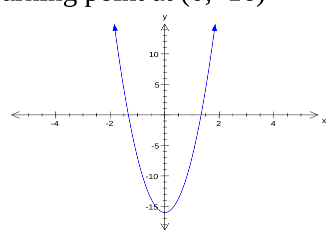


g $y = 9x^2 - 16 = (3x - 4)(3x + 4)$

i (0,–16), $(\frac{4}{3}$,0) and $(-\frac{4}{3}$,0)

ii $x = -\frac{b}{2a} = 0$

iii Turning point at (0,–16)

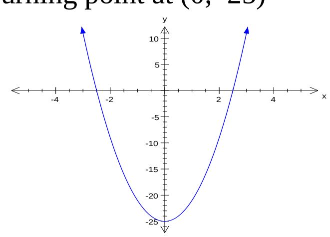


h $y = 4x^2 - 25 = (2x - 5)(2x + 5)$

i (0,–25), $(\frac{5}{2}$,0) and $(-\frac{5}{2}$,0)

ii $x = -\frac{b}{2a} = 0$

iii Turning point at (0,–25)



7 $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a $x^2 + 6x + 3 = 0$

$\therefore x = \dfrac{-6 \pm \sqrt{36 - 12}}{2}$

$= \dfrac{-6 \pm 4\sqrt{6}}{2} = -3 \pm 2\sqrt{6}$

$x = -0.55, -5.45$ from calculator

b $x^2 + 9x + 12 = 0$

$\therefore x = \dfrac{-9 \pm \sqrt{81 - 48}}{2}$

$= \dfrac{-9 \pm \sqrt{33}}{2}$

$x = -1.63, -7.37$ from calculator

c $x^2 - 4x + 2 = 0$

$\therefore x = \dfrac{4 \pm \sqrt{16 - 8}}{2}$

$= 2 \pm \sqrt{2}$

$x = 3.414, 0.586$ from calculator

d $2x^2 + 7x + 2 = 0$

$\therefore x = \dfrac{-7 \pm \sqrt{49 - 16}}{2}$

$= \dfrac{-7 \pm \sqrt{33}}{2}$

$x = -0.314, -3.186$ from calculator

e $2x^2 + 7x + 4 = 0$

$\therefore x = \dfrac{-7 \pm \sqrt{49 - 32}}{2}$

$= \dfrac{-7 \pm \sqrt{17}}{2}$

$x = -0.719, -2.7816$ from calculator

f $3x^2 + 9x - 1 = 0$

$\therefore x = \dfrac{-9 \pm \sqrt{81 + 12}}{2}$

$= \dfrac{-9 \pm \sqrt{93}}{2}$

$x = -0.107, 3.107$ from calculator

8 $y = a(x - b)(x - c)$
Assume the graph cuts the axis at (0,0) and (5,0), $b = 0$ and $c = 5$
Using (6, 10): $y = ax(x - 5) = 10$

$\therefore\ ax^2 - 5ax - 10 = 0$

$36a - 30a - 10 = 0$

$6a - 10 = 0$

$\therefore\ a = \dfrac{5}{3}$

$y = \dfrac{5}{3}x(x - 5)$

9 A parabola has the same shape as $y = 3x^2$, but its vertex is at (5,2).

$\therefore y = 3(x - 5)^2 + 2$

10 The vertex is at (1,5).

$\therefore y = a(x - 1)^2 + 5$

Using (2, 10):

$y = a(2 - 1)^2 + 5 = 10$

$\therefore a = 5$

$\therefore y = 5(x - 1)^2 + 5$

OR $y = 5x^2 - 10x + 10$

11

a $y = 2x + 3$ and $y = x^2$ meet where:

$x^2 = 2x + 3,\ \therefore x^2 - 2x - 3 = 0$

$\therefore (x - 3)(x + 1) = 0$

Where $x = 3$, $y = 9$; where $x = -1$, $y = 1$
Curves meet at (3,9) and (–1,1).

b $y = 8x + 11$ and $y = 2x^2$ meet where:

$2x^2 = 8x + 11$

$\therefore\ 2x^2 - 8x - 11 = 0$

$\therefore x = \dfrac{8 \pm \sqrt{64 + 88}}{4}$

$\therefore x = 2 \pm \dfrac{\sqrt{38}}{2}$

Where $x = 2 - \dfrac{\sqrt{38}}{2}$, $y = 27 - 4\sqrt{38}$

Where $x = 2 + \dfrac{\sqrt{38}}{2}$, $y = 27 + 4\sqrt{38}$

From calculator: curves meet at (–1.08,2.34) and (5.08,51.66).

c $y = 3x^2 + 7x$ and $y = 2$ meet where:

$3x^2 + 7x = 2$

$\therefore 3x^2 + 7x - 2 = 0$

$\therefore x = \dfrac{-7 \pm \sqrt{49 - 24}}{6}$

$\therefore x = \dfrac{-7 \pm \sqrt{73}}{6}$

Curves meet at $\left(\frac{-7 \pm \sqrt{73}}{6}, 2\right)$.

From calculator: (0.26, 2) and (–2.62,2)

d $y = 2x^2$ and $y = 2 - 3x$ meet where

$2x^2 = 2 - 3x$

$\therefore 2x^2 + 3x - 2 = 0$

$\therefore (2x - 1)(x + 2) = 0, \therefore x = \dfrac{1}{2}, -2$

Where $x = \frac{1}{2}$, $y = \frac{1}{2}$; where $x = -2$, $y = 8$

Curves meet at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and (–2,8).

12

a $2x^2 + mx + 1 = 0$ has exactly one solution where $\Delta = 0$:

$\Delta = m^2 - 8 = 0, \therefore m^2 = 8$

$\therefore m = \pm 2\sqrt{2}$

b $x^2 - 4mx + 20 = 0$ has real solutions where $\Delta \geq 0$:

$\Delta = 16m^2 - 80 \geq 0$

$\therefore m^2 \geq 5$

Solution set:

$\{m: m \leq -\sqrt{5}\} \cap \{m: m \geq \sqrt{5}\}$

c $4mx^2 + 4(m - 1)x + m - 2 = 0$

There are real solutions if $\Delta \geq 0$:

$\Delta = 16(m - 1)^2 - 16m(m - 2)$

$= 16(m^2 - 2m + 1 - m^2 + 2m)$

$= 16 > 0$

Therefore there are 2 distinct real solutions for all values of m.

Chapter Review: Extended-response Solutions

1 **a** The turning point (h, k) is $\left(25, \frac{9}{2}\right)$

$\therefore \quad y = a(x - 25)^2 + \frac{9}{2}$

When $x = 0$, $\quad y = 0$

$\therefore \quad 0 = a(0 - 25)^2 + \frac{9}{2}$

$\therefore \quad 0 = 625a + \frac{9}{2}$

$\therefore \quad 625a = \frac{-9}{2}$

$\therefore \quad a = \frac{-9}{1250}$

Hence the equation for the parabola is $y = \frac{-9}{1250}(x - 25)^2 + \frac{9}{2}$, for $0 \le x \le 50$.

This can also be written as $y = -0.0072x(x - 50)$ [the intercept form].

b

x	0	5	10	15	20	25	30	35	40	45	50
y	0	1.62	2.88	3.78	4.32	4.5	4.32	3.78	2.88	1.62	0

You can find these values using a CAS calculator, or:

When $x = 10$, $\quad y = \frac{-9}{1250}(10 - 25)^2 + \frac{9}{2}$

$= \frac{-9}{1250} \times 225 + \frac{9}{2}$

$= \frac{-81}{50} + \frac{225}{50}$

$= \frac{144}{50}$

$= \frac{72}{25}$

$= 2.88$

When $x = 20$, $\quad y = \frac{-9}{1250}(20 - 25)^2 + \frac{9}{2}$

$= \frac{-9}{1250} \times 25 + \frac{9}{2}$

$= 4.32$

When $x = 30$, $\quad y = \frac{-9}{1250}(30 - 25)^2 + \frac{9}{2}$

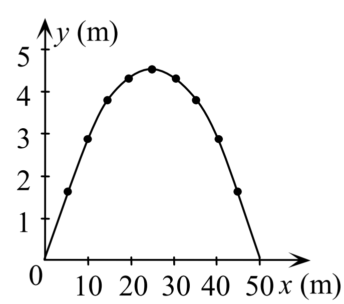
$= \frac{-9}{1250} \times 25 + \frac{9}{2}$

$= 4.32$

When $x = 40$, $\quad y = \frac{-9}{1250}(40 - 25)^2 + \frac{9}{2}$

$= \frac{-9}{1250} \times 225 + \frac{9}{2}$

$= 2.88$



c When $y = 3$, $\frac{-9}{1250}(x - 25)^2 + \frac{9}{2} = 3$

$\therefore \quad \frac{-9}{1250}(x - 25)^2 = \frac{-3}{2}$

$\therefore \quad (x - 25)^2 = \frac{-3}{2} \times \frac{-1250}{9} = \frac{625}{3}$

$\therefore \quad x - 25 = \pm\sqrt{\frac{625}{3}}$

$\therefore \quad x = 25 \pm \frac{25\sqrt{3}}{3}$

$\therefore \quad x \approx 10.57$ or $x \approx 39.43$

Hence the height of the arch is 3 m above water level approximately 10.57 m and 39.43 m horizontally from A. This can also be solved using a CAS calculator.

d When $x = 12$, $y = \frac{-9}{1250}(12 - 25)^2 + \frac{9}{2}$

$= \frac{-9}{1250} \times 169 + \frac{9}{2} = 3.2832$

The height of the arch is 3.2832 m at a horizontal distance of 12 m from A.

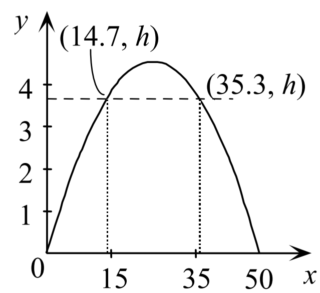
e The greatest height of the deck above water level, h m, is when $x + 0.3 = 15$ and $x - 0.3 = 35$

i.e. when $x = 14.7$ and $x = 35.3$

$\therefore \quad h = \frac{-9}{1250}(14.7 - 25)^2 + \frac{9}{2}$

$= 3.736152$

Hence the greatest height of the deck above water level is approximately 3.736 m.



2 **a** If x cm is the side length of the square then $4x$ cm has been used to form the square, so the perimeter of the rectangle is $P = 12 - 4x$.
Let a cm be the width of the rectangle and $2a$ cm be the length of the rectangle,
so $P = a + a + 2a + 2a = 6a$

$\therefore \quad 6a = 12 - 4x$

$\therefore \quad a = 2 - \frac{2}{3}x$ and $2a = 4 - \frac{4}{3}x$

Hence the dimensions of the rectangle are $\left(2 - \frac{2}{3}x\right)$ cm $\times \left(4 - \frac{4}{3}x\right)$ cm.

b Let A_1 be the area of the square and A_2 be the area of the rectangle.

$\therefore \quad A = A_1 + A_2 = x^2 + \left(2 - \frac{2}{3}x\right)\left(4 - \frac{4}{3}x\right)$

$= x^2 + 8 - \frac{8}{3}x - \frac{8}{3}x + \frac{8}{9}x^2$

$$= \frac{17}{9}x^2 - \frac{16}{3}x + 8$$

Hence the combined area of the square and the rectangle in cm^2 is defined by the rule $A = \frac{17}{9}x^2 - \frac{16}{3}x + 8$.

c TP occurs when $x = -\frac{b}{2a}$

$$= \frac{16}{3} \div \frac{34}{9}$$

$$= \frac{24}{17}$$

Minimum occurs when $x = \frac{24}{17}$.

When $x = \frac{24}{17}$, $\quad 4x = \frac{96}{17} \approx 5.65$

and $\quad 12 - 4x = \frac{108}{17} \approx 6.35$

Hence, the wire needs to be cut into lengths of 5.65 cm and 6.35 cm (correct to 2 decimal places) for the sum of the areas to be a minimum.

3 a V = rate × time

When $x = 5$, $V = 0.2 \times 60 = 12$

When $x = 10$, $V = 0.2 \times 60 \times 5 = 60$

When $x = 0$, $V = 0$

$\therefore$ $c = 0$ (y-axis intercept is 0)

$\therefore$ $V = ax^2 + bx$

When $x = 5$, $V = 12$, $\quad 12 = 25a + 5b \quad (1)$

When $x = 10$, $V = 60$, $\quad 60 = 100a + 10b \quad (2)$

$2 \times (1)$ $\quad 24 = 50a + 10b \quad (3)$

$(2) - (3)$ $\quad 36 = 50a$

$\therefore$ $\quad a = \frac{36}{50} = \frac{18}{25}$

Substitute $a = \frac{18}{25}$ in (1) $\quad 12 = 25 \times \frac{18}{25} + 5b$

$\therefore$ $\quad 12 = 18 + 5b$

$\therefore$ $\quad 5b = -6$

$\therefore$ $\quad b = -\frac{6}{5}$

Hence, the rule for V in terms of x is $V = \frac{18}{25}x^2 - \frac{6}{5}x$, $x \geq 0$, or $V = 0.72x^2 - 1.2x$

b When $x = 20$ (i.e. a depth of 20 cm),

$$V = \frac{18}{25}(20)^2 - \frac{6}{5}(20)$$

$$= \frac{18 \times 400}{25} - 24$$

$$= 18 \times 16 - 24 = 264$$

Now $\quad V$ = rate × time

$\therefore$ time $= \dfrac{V}{\text{rate}}$

$= \dfrac{264}{0.2} = 1320$ minutes

$= 22$ hours

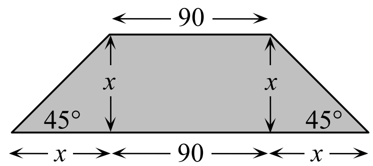
Water can be pumped into the tank for 22 hours before overflowing.

4 a Let V_E m^3 be the volume of the embankment.

$V_E = 120 \times$ shaded area

$= 120\left(90x + \frac{1}{2}x^2 + \frac{1}{2}x^2\right)$

$= 120x^2 + 10\,800x,\ x > 0$



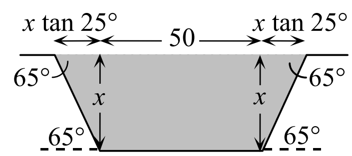
b Let V_C m^3 be the volume of the cutting.

$V_C = 100 \times$ shaded area

$= 100(50x + x^2 \tan 25°)$

$\approx 100(50x + 0.466\,308x^2)$

$\approx 46.63x^2 + 5000x,\ x > 0$



c When $x = 4$, $V_C \approx 46.63 \times 4^2 + 5000 \times 4$

$\approx 20\,746.08$

Now $V_E = L \times (x^2 + 90x)$, where L m is the length of the embankment.

$\therefore$ $L = \dfrac{V_E}{x^2 + 90x}$

If using soil from the cutting, $V_C = V_E$

$\therefore$ $L = \dfrac{V_C}{x^2 + 90x}$

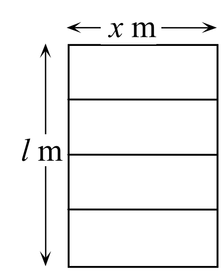
$= \dfrac{20\,746.08}{4^2 + 90 \times 4} \approx 55.18$

Hence, when $x = 4$ m, an embankment 55.18 m long could be constructed from the soil taken from the cutting.

5 a $5x + 2l = 100$

$\therefore$ $2l = 100 - 5x$

$\therefore$ $l = 50 - \frac{5}{2}x$



b $A = x \times l$

$= 50x - \frac{5}{2}x^2$

c When $A = 0$, $-\frac{5}{2}x^2 + 50x = 0$

$\therefore$ $x\left(-\frac{5}{2}x + 50\right) = 0$

$\therefore$ either $x = 0$ or $-\frac{5}{2}x + 50 = 0$

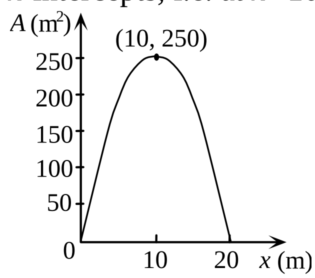
If $-\frac{5}{2}x + 50 = 0$, $\quad \frac{5}{2}x = 50$

$$\therefore \quad x = \frac{2 \times 50}{5} = 20$$

Turning point is halfway between the x-intercepts, i.e. at x=10.

When $x = 10$,

$$A = -\frac{5}{2} \times 10^2 + 50 \times 10 = -250 + 500 = 250$$



Completing the square may also be used to find the vertex.

d The maximum area is 250 m² when x is 10 metres.

6 Given $AP = 1$, $AB = 1 - x$, $AD = x$ and $\frac{AP}{AD} = \frac{AD}{AB}$

then $\frac{1}{x} = \frac{x}{1-x}$

$\therefore \quad 1 - x = x^2$

$\therefore \quad x^2 + x - 1 = 0$

Using the general quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where } a = 1, b = 1, c = -1$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

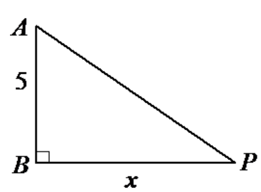
$$= \frac{-1 - \sqrt{5}}{2} \text{ or } \frac{-1 + \sqrt{5}}{2}$$

but $x > 0$, so $x = \frac{-1 + \sqrt{5}}{2}$

7 **a** Using Pythagoras' theorem

$$PA^2 = 5^2 + x^2 = x^2 + 25$$

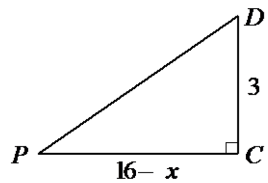
$$\therefore \quad PA = \sqrt{x^2 + 25}$$



b **i** $PC = BC - BP = 16 - x$

ii Using Pythagoras' theorem

$$PD^2 = (16 - x)^2 + 3^2$$
$$= x^2 - 32x + 256 + 9$$
$$\therefore \quad PD = \sqrt{x^2 - 32x + 265}$$



c If $PA = PD$,

$$\sqrt{x^2 + 25} = \sqrt{x^2 - 32x + 265}$$
$$\therefore \quad x^2 + 25 = x^2 - 32x + 265$$
$$\therefore \quad 25 = -32x + 265$$
$$\therefore \quad 32x = 240$$
$$\therefore \quad x = 7.5$$

d If $PA = 2PD$,

$$\sqrt{x^2 + 25} = 2\sqrt{x^2 - 32x + 265}$$
$$\therefore \quad x^2 + 25 = 4(x^2 - 32x + 265)$$
$$= 4x^2 - 128x + 1060$$
$$\therefore \quad 3x^2 - 128x + 1035 = 0$$

Using the general quadratic formula,

$$x = \frac{128 \pm \sqrt{(-128)^2 - 4(3)(1035)}}{2(3)}$$
$$= \frac{128 \pm \sqrt{3964}}{6}$$
$$= \frac{128 \pm 2\sqrt{991}}{6}$$
$$= \frac{64 \pm \sqrt{991}}{3}$$
$$= 31.826\,71\ldots \text{ or } 10.839\,94\ldots$$
$$\approx 10.840 \text{ (as } 0 \le x \le 16)$$

e If $PA = 3PD$,

$$\sqrt{x^2 + 25} = 3\sqrt{x^2 - 32x + 265}$$
$$\therefore \quad x^2 + 25 = 9(x^2 - 32x + 265)$$
$$= 9x^2 - 288x + 2385$$
$$\therefore \quad 8x^2 - 288x + 2360 = 0$$
$$\therefore \quad 8(x^2 - 36x + 295) = 0$$

Using the general quadratic formula,

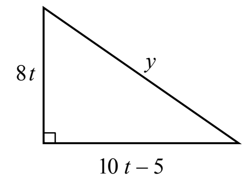
$$x = \frac{36 \pm \sqrt{(-36)^2 - 4(1)(295)}}{2(1)}$$
$$= \frac{36 \pm \sqrt{116}}{2}$$
$$= \frac{36 \pm 2\sqrt{29}}{2} = 18 \pm \sqrt{29}$$
$$= 23.385\,16\ldots \text{ or } 12.614\,83\ldots$$
$$\approx 12.615 \text{ (as } 0 \le x \le 16)$$

Note: Parts **c**, **d** and **e** can be solved using the CAS calculator. Plot the graphs of ***f1*****=√(*****x*****^2+25)**, ***f2*****=√(*****x*****^2–32*****x*****+265)**, ***f3*****=2×f2(*****x*****)** and ***f4*****=3×f2(*****x*****)** for $x \in [0, 16]$. The points of intersection of ***f1*** with each of the other graphs provide the solutions for *x*.

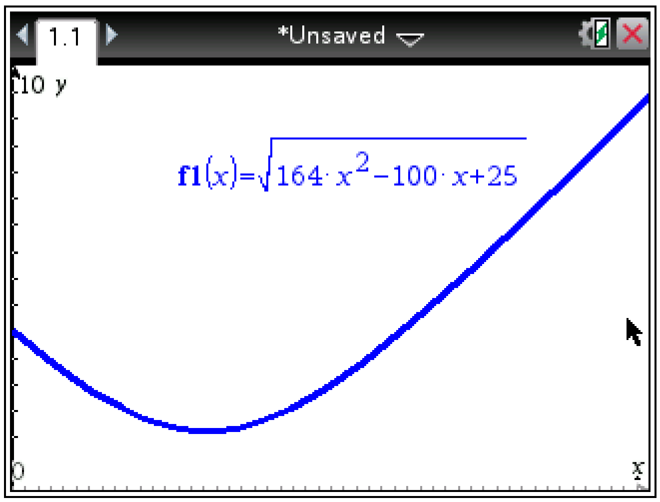
8 a i Consider AB and CD to be a pair of Fartesian axes with O at the point (0, 0). The first jogger is at the point $(8t, 0)$ at time t. The second jogger is at the point $(0, 10t - 5)$ at time t.

Using Pythagoras' theorem

$$y^2 = (8t)^2 + (10t - 5)^2$$
$$= 64t^2 + 100t^2 - 100t + 25$$
$$\therefore \quad y = \sqrt{164t^2 - 100t + 25}$$



ii



iii On a CAS calculator, enter **solve(√(164x^2–100x+25)=4,x)**.

The points of intersection are $\left(\frac{9}{82}, 4\right)$ and $\left(\frac{1}{2}, 4\right)$.

Therefore the joggers are 4 km apart after 0.11 hours (1.07 pm), correct to 2 decimal places, and after 0.5 hours (1.30 pm).

Or consider $\sqrt{164t^2 - 100t + 25} = 4$

$$\therefore \quad 164t^2 - 100t + 9 = 0$$

$$\therefore \quad t = \frac{100 \pm \sqrt{(-100)^2 - 4(9)(164)}}{2(164)}$$
$$= \frac{100 \pm \sqrt{4096}}{328}$$
$$= \frac{100 \pm 64}{328}$$
$$= \frac{1}{2} \text{ or } \frac{9}{82}$$

iv With the graph from part **ii** on screen

TI: Press **Menu→6:Analyze Graph→2:Minimum**

CP: Tap **Analysis→G-Solve→Min**

to yield (0.304 878 37, 3.123 475 2).

Therefore the joggers are closest when they are 3.12 km apart after 0.30 hours, correct to 2 decimal places.

Alternatively, the minimum of $\sqrt{164t^2 - 100t + 25}$ occurs when

$164t^2 - 100t + 25$ is a minimum.

This occurs when $t = \dfrac{100}{2 \times 164}$

$= \dfrac{25}{82}$ (1.18 pm)

$\therefore$ minimum distance apart $= \dfrac{20}{\sqrt{41}}$

$= \dfrac{20\sqrt{41}}{41}$

≈ 3.123 km

b **i** When $y = 5$, $5 = \sqrt{164t^2 - 100t + 25}$

$\therefore$ $25 = 164t^2 - 100t + 25$

$\therefore$ $164t^2 - 100t = 0$

$\therefore$ $4t(41t^2 - 25t) = 0$

$\therefore$ $t = 0$ or $t = \dfrac{25}{41}$

ii When $y = 6$, $6 = \sqrt{164t^2 - 100t + 25}$

$\therefore$ $36 = 164t^2 - 100t + 25$

$\therefore$ $164t^2 - 100t - 11 = 0$

Using the general quadratic formula,

$$t = \frac{100 \pm \sqrt{(-100)^2 - 4(164)(-11)}}{2(164)}$$

$$= \frac{100 \pm \sqrt{17\,216}}{328} = \frac{25 \pm 2\sqrt{269}}{82}$$

9 **a** $BC = x$, $CD = y$, $BD =$ diameter of circle $= 2a$

Using Pythagoras' theorem,

$BC^2 + CD^2 = BD^2$

$\therefore$ $x^2 + y^2 = 4a^2$, as required.

b Perimeter $= b$, but perimeter $= 2(x + y)$

$\therefore$ $2(x + y) = b$

c $2(x + y) = b$ $\therefore$ $2x + 2y = b$

$\therefore$ $2y = b - 2x$

$\therefore$ $y = \dfrac{1}{2}b - x$ (1)

Substituting (1) into $x^2 + y^2 = 4a^2$ gives

$$x^2 + \left(\frac{1}{2}b - x\right)^2 = 4a^2$$

$\therefore$ $2x^2 - bx + \dfrac{1}{4}b^2 - 4a^2 = 0$ (2)

$\therefore$ $8x^2 - 4bx + b^2 - 16a^2 = 0$

d Now $x + y > 2a$ $\therefore$ using (1), $x + \left(\dfrac{1}{2}b - x\right) > 2a$

$\therefore \quad \frac{1}{2}b > 2a$

$\therefore \quad b > 4a$

Considering the discriminant, Δ, of (2)

$$\begin{aligned}\Delta &= (-b)^2 - 4(2)\left(\frac{1}{4}b^2 - 4a^2\right)\\ &= b^2 - 8\left(\frac{1}{4}b^2 - 4a^2\right)\\ &= b^2 - 2b^2 + 32a^2\\ &= 32a^2 - b^2\end{aligned}$$

For the inscribed rectangle to exist, $\Delta \geq 0$

$\therefore \quad 32a^2 - b^2 \geq 0$

$\therefore \quad b^2 \leq 32a^2$

$\therefore \quad b \leq 4\sqrt{2}a$

$\therefore$ $4a < b \leq 4\sqrt{2}a$, as required.

e **i** Substituting $a = 5$ and $b = 24$ into (2) gives

$$2x^2 - 24x + \left(\frac{1}{4}(24)^2 - 4(5)^2\right) = 0$$

$\therefore \quad 2x^2 - 24x + 44 = 0$

$\therefore \quad x^2 - 12x + 22 = 0$

Using the general quadratic formula,

$$\begin{aligned}x &= \frac{12 \pm \sqrt{(-12)^2 - 4(1)(22)}}{2(1)}\\ &= \frac{12 \pm \sqrt{56}}{2}\\ &= 6 \pm \sqrt{14}\end{aligned}$$

Now $\quad y = \frac{1}{2}b - x$

$\quad = \frac{1}{2}(24) - x = 12 - x$

When $x = 6 \pm \sqrt{14}$, $\quad y = 12 - (6 \pm \sqrt{14})$

When $x = 6 + \sqrt{14}$, $\quad y = 6 - \sqrt{14}$

When $x = 6 - \sqrt{14}$, $\quad y = 6 + \sqrt{14}$

ii If $b = 4\sqrt{2}a$, then (2) gives

$$2x^2 - 4\sqrt{2}ax + \left(\frac{1}{4}(4\sqrt{2}a)^2 - 4a^2\right) = 0$$

$\therefore \quad 2x^2 - 4\sqrt{2}ax + 8a^2 - 4a^2 = 0$

$\therefore \quad 2x^2 - 4\sqrt{2}ax + 4a^2 = 0$

$\therefore \quad x^2 - 2\sqrt{2}ax + 2a^2 = 0$

$\therefore \quad (x - \sqrt{2}a)^2 = 0$

$\therefore \quad x = \sqrt{2}a$

$\therefore \quad y = \frac{1}{2}b - x$

$\quad = 2\sqrt{2}a - \sqrt{2}a$

$\quad = \sqrt{2}a$

f If $\frac{b}{a} = 5$, then $b = 5a$ and, from (2):

$$2x^2 - (5a)x + \left(\frac{1}{4}(5a)^2 - 4a^2\right) = 0$$

$$\therefore \quad 2x^2 - 5ax + \left(\frac{25}{4}a^2 - 4a^2\right) = 0$$

$$\therefore \quad 2x^2 - 5ax + \frac{9}{4}a^2 = 0$$

Using the general quadratic formula,

$$x = \frac{5a \pm \sqrt{(-5a)^2 - 4 \times 2 \times \frac{9}{4}a^2}}{2(2)}$$

$$= \frac{5a \pm \sqrt{25a^2 - 18a^2}}{4}$$

$$= \frac{5a \pm \sqrt{7}a}{4}$$

Now $\quad y = \frac{1}{2}b - x$

$$= \frac{1}{2}(5a) - x$$

$$= \frac{5}{2}a - x$$

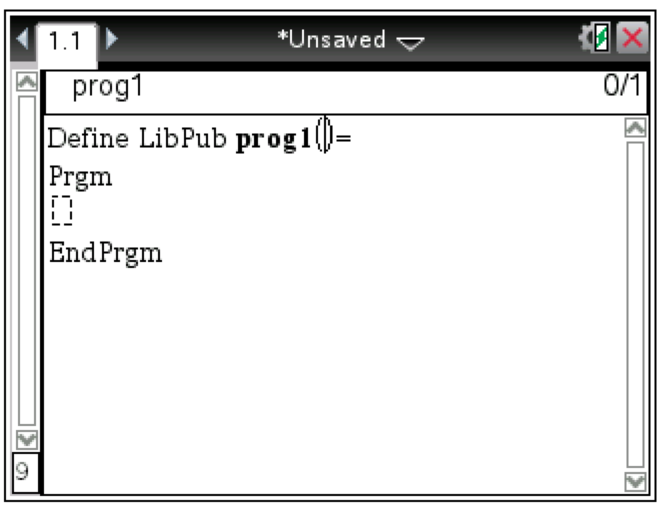
When $x = \frac{5a \pm \sqrt{7}a}{4}$, $\quad y = \frac{5}{2}a - \frac{5a \pm \sqrt{7}a}{4}$

When $x = \frac{5a + \sqrt{7}a}{4}$, $\quad y = \frac{5a - \sqrt{7}a}{4}$

When $x = \frac{5a - \sqrt{7}a}{4}$, $\quad y = \frac{5a + \sqrt{7}a}{4}$

g The following program can be input into a CAS calculator to solve equation (2) in part **c** for x and y, given a and b ($a, b \in R$), correct to 2 decimal places.

TI: In the calculator application press **menu→9:Functions & Programs→1:Program Editor→1:New**. Name the program prog1. The following information is shown automatically. Complete the screen as follows:Complete the screen as follows:



```
Define LibPub prog1()=
Prgm
setMode(5,2)
setMode(1,16)
Local a,b,w,x,y,z
Request "a = ",a
Request "b = ",b
(b+√(32a^2–b^2))/4 → x
b/2–x → y
(b–√(32a^2–b^2))/4 → w
b/2–w → z
Disp "x = ", x
Disp "and y = ", y
Disp "OR"
Disp "x=",w
Disp "and y =",z
EndPrgm
```

10 a Equation of curve A is

$y = (x - h)^2 + 3$

(0,4): $4 = (0 - h)^2 + 3$

$h^2 = 1$

h=1 (since h>0)

So $y = (x - 1)^2 + 3$

$= x^2 - 2x + 4$

Giving b= -2, c=4 and h=1

b **i** The coordinates of P' are $(x, -6 + 4x - x^2)$

ii Let (m, n) be the coordinates of M.

$\therefore \quad m = x$

and $\quad n = \dfrac{(x^2 - 2x + 4) + (-6 + 4x - x^2)}{2}$

$= \dfrac{2x - 2}{2} = x - 1$

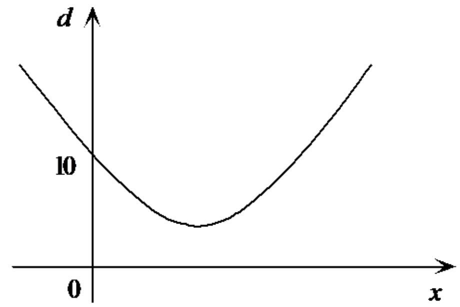
$\therefore$ the coordinates of M are $(x, x - 1)$.

iii The coordinates of M for $x = 0, 1, 2, 3, 4$ are (0, –1), (1, 0), (2, 1), (3, 2) and (4, 3) respectively.

iv $y = x - 1$ is the equation of the straight line on which the points (0, –1), (1, 0), (2, 1), (3, 2) and (4, 3) all lie.

c **i** $d = (x^2 - 2x + 4) - (-6 + 4x - x^2) = 2x^2 - 6x + 10$

ii



iii **TI:** Press **Menu→6:Analyze Graph→2:Minimum** with ***f*1=2*x*^2–6*x*+10**,
CP: Tap **Analysis→G-Solve→Min**
to yield (1.5, 5.5). Therefore the minimum value of d is 5.5 and this occurs when $x = 1.5$.

Or consider $2(x^2 - 3x + 5) = 2\left(x^2 - 3x + \frac{9}{4} + 5 - \frac{9}{4}\right)$

$$= 2\left(\left(x - \frac{3}{2}\right)^2 + \frac{11}{4}\right) = 2\left(x - \frac{3}{2}\right)^2 + \frac{11}{2}$$

$\therefore$ minimum value of d is $\frac{11}{2}$ and occurs for $x = \frac{3}{2}$.

11 a Length of path $= \sqrt{(60 + 30)^2 + (30 + 15)^2}$

$$= \sqrt{10\,125}$$
$$= 45\sqrt{5}$$

b i $y = ax^2 + bx + c$

At (–20, 45),	$45 = 400a - 20b + c$	(1)
At (40, 40),	$40 = 1600a + 40b + c$	(2)
At (30, 35),	$35 = 900a + 30b + c$	(3)
(2) – (1) gives	$-5 = 1200a + 60b$	(4)
(2) – (3) gives	$5 = 700a + 10b$	(5)
6 × (5) – (4) gives	$35 = 3000a$	

$\therefore \quad a = \frac{35}{3000}$

$$= \frac{7}{600}$$

Substituting $a = \frac{7}{600}$ into (5) gives:

$$5 = 700\left(\frac{7}{600}\right) + 10b$$
$$= \frac{49}{6} + 10b$$

$\therefore \quad 10b = \frac{-19}{6}$

$\therefore \quad b = \frac{-19}{60}$

Substituting $a = \frac{7}{600}$ and $b = \frac{-19}{60}$ into (1) gives:

$$45 = 400\left(\frac{7}{600}\right) - 20\left(\frac{-19}{60}\right) + c$$

$$= \frac{14}{3} + \frac{19}{3} + c$$

$\therefore \quad c = 34$

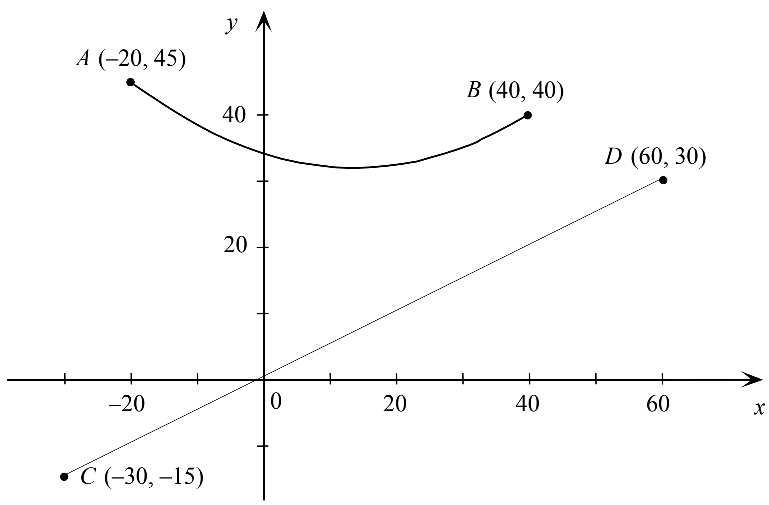
$\therefore \quad y = \frac{7}{600}x^2 - \frac{19}{60}x + 34$

ii **TI:** Press **Menu→6:Analyze Graph→2:Minimum** with ***f*1=7/600*x*^2–19/60*x*+34**,
CP: Tap **Analysis→G-Solve→Min**
to yield (13.571 521, 31.851 19). Therefore the vertex of the parabola has coordinates (13.57, 31.85), correct to 2 decimal places.

Or consider
$$\frac{7}{600}x^2 - \frac{19}{60}x + 34 = \frac{7}{600}\left(x^2 - \frac{190}{7}x + \frac{20\,400}{7}\right)$$
$$= \frac{7}{600}\left(\left(x - \frac{95}{7}\right)^2 + \frac{133\,775}{49}\right)$$
$$= \frac{7}{600}\left(x - \frac{95}{7}\right)^2 + \frac{5351}{168}$$

$\therefore$ minimum value is $\frac{5351}{168}$ and this occurs when $x = \frac{95}{7}$.

c



d **i** The expression $y = (ax^2 + bx + c) - \frac{1}{2}x$ determines the distance, perpendicular to the x-axis, between $y = ax^2 + bx + c$ and $y = \frac{1}{2}x$ at the point x. In this question, it is the distance between the path and the pond.

ii With ***f1*=7*x*^2/600–19*x*/60+34–*x*/2**
TI: Press **Menu→6:Analyze Graph→2:Minimum**
CP: Tap **Analysis→G-Solve→Min**
to yield (35.000 004, 19.708 333). Therefore the minimum value is 19.71 which occurs when x = 35.00, correct to 2 decimal places.

Or consider
$$\frac{7x^2}{600} - \frac{19x}{60} + 34 - \frac{x}{2} = \frac{7x^2}{600} - \frac{49x}{60} + 34$$
$$= \frac{7}{600}\left(x^2 - 70x + \frac{20\,400}{7}\right)$$

$$= \frac{7}{600}\left(x^2 - 70x + 1225 + \frac{11\,825}{7}\right)$$

$$= \frac{7}{600}\left((x - 35)^2 + \frac{11\,825}{7}\right)$$

$$= \frac{7}{600}(x - 35)^2 + \frac{473}{24}$$

$\therefore$ minimum value is $\frac{473}{24}$ which occurs when $x = 35$.