Worked solutions

Chapter 4 Aspects of motion

4.1 Describing motion in a straight line

1 **a** Displacement = 40 – 0 = +40 cm, distance = 40 cm
b Displacement = 40 – 50 = –10 cm, distance = 10 cm
c Displacement = 70 – 50 = +20 cm, distance = 20 cm
d Displacement = 70 – 50 = +20 cm, distance = 80 cm

2 **a** Distance = 50 + 30 = 80 km
b Displacement = 30 – 50 = –20 km or 20 km north

3 **a** 10 m down
b 60 m up
c distance = 10 + 60 = 70 m
d 50 m up (take ground floor = 0 displacement)

4 Displacement, as it is the only one of the physical quantities that must have a direction.

5 Let north be positive.
a –20 + 10 = –10 m = D
b same as part **a**, answer = D
c 3(10) = 30 m = C
d –(–20) =+20 m = A

6 **a** Distance = 10 + 4 + 15 + 5 + 5 = 39 steps
b 1 step west of the clothesline.
c 1 step west of the clothesline.

7 **a** ≈2 m s^{-1}
b ≈1 mm s^{-1}
c ≈10 m s^{-1}
d ≈5 m s^{-1}

8 **a** Average speed = 2.5/(10/60) = 15 km h^{-1}
b 15/3.6 = 4.2 m s^{-1}
c No. She would probably be travelling faster or slower than the average speed most of the time, depending on the traffic and the terrain.

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9 **a** Average speed = 400/18.0 = 22.2 m s^{-1}

b $a = \dfrac{120-0}{18.0}$ = 6.7 km h^{-1} s^{-1}

c a = 6.667/3.6 = 1.85 m s^{-2}

d v = 120/3.6 = 33.33 m s^{-1}
Distance = 33.33 × 0.60 = 20 m

10 **a** Change in speed = $v - u$ = 15 – 25 = –10 m s^{-1}

b Change in velocity = $v - u$ = 15 m s^{-1} west – 25 m s^{-1} east
= 15 m s^{-1} west + 25 m s^{-1} west = 40 m s^{-1} west

c Magnitude of acceleration = (change in velocity)/time taken
= (40 m s^{-1} west)/0.050 s = 800 m s^{-2} (direction not required)

11 **a** magnitude of acceleration = $\dfrac{0-60\,\text{km h}^{-1}}{5.0\,\text{s}}$ = –12 km h^{-1} s^{-1} or 12 km h^{-1} s^{-1}
(direction not required)

b (12 km h^{-1} s^{-1} south)/3.6 = 3.3 m s^{-2} south

12 **a** $d = v \times t$ = 30 × 50 = 1500 m

b Average speed = d/t = 1500/878 = 1.7 m s^{-1}

c He finished at the position at which he started, therefore his displacement is 0.

d Average velocity = displacement/time = 0

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4.2 Graphing motion: position, velocity and acceleration

1 **a** +40 cm, assuming movement to the right is positive.

b The dancer is at rest in those sections where the gradient of the position–time graph is zero, i.e. sections A and C.

c The dancer is moving in a positive direction in the sections with a positive gradient, i.e. section B.

d A negative velocity is indicated by the section with a negative gradient, i.e. section D.

e Average speed = distance travelled/time taken = 8/20 = 0.40 m s^{-1}

2 The car initially moves in a positive direction and travels 8 m in 2 s. It then stops for 2 s. The car then reverses direction for 5 s, passing back through its starting point after 8 s. It travels a further 2 m in a negative direction before stopping after 9 s.

3 Reading from graph:

a +8 m

b +8 m

c +4 m

d –2 m

4 The car returns to its starting point when its position is zero again, i.e. at t = 8 s.

5 **a** The velocity during the first 2 s is equal to the gradient of the graph during this interval.
Velocity = rise/run = (8 – 0)/2 = +4 m s^{-1}

b After 3 s the velocity is zero, since the gradient of the graph = 0.

c The velocity = gradient of graph
= rise/run = (0 – 8)/4 = –2 m s^{-1}

d The velocity at 8 s is = –2 m s^{-1}, since the car is travelling at a constant velocity of –2 m s^{-1} between 4 s and 9 s.

e The velocity from 8 s to 9 s = –2 m s^{-1}, since the car is travelling at a constant velocity of –2 m s^{-1} between 4 s and 9 s.

6 **a** Distance = 8 + 8 + 2 = 18 m

b Displacement =Δx = (–2) – 0 = –2 m.

7 **a** The cyclist travels with a constant velocity in a positive direction for the first 30 s, travelling 150 m during this time. Then the cyclist speeds up for 10 s, travelling a further 150 m. The cyclist maintains this increased speed for the final 10 s, covering another 200 m in this time.

b Velocity = rise/run = (150 – 0)/30 = +5 m s^{-1}

c Velocity = rise/run =(500 – 300)/10 s = +20 m s^{-1}

d The instantaneous velocity at 35 s is found from the gradient of the tangent to the curve at t = 35 s, and is 13 m s^{-1}.

e The average velocity between 30 s and 40 s
= $\Delta x/\Delta t$ = (300 – 150)/10 = 15 m s^{-1}

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8 **a** B. $\Delta v/\Delta t$ is constant and negative.

b A. $\Delta v/\Delta t = 0$.

c C. $\Delta v/\Delta t$ is constant and positive.

d D. $\Delta v/\Delta t$ is positive and decreasing as time increases.

9 **a** Running north at 1 m s^{-1}.

b Increasing speed from 1 m s^{-1} to 3 m s^{-1} while running north.

c Running north but slowing to a stop.

d Stationary

e Accelerating from rest to 1 m s^{-1} while running south.

f Running south at 1 m s^{-1}.

10 **a** Displacement is found from the area under the *v–t* graph from $t = 0$ s to $t = 2$:
$s = 2$ m north

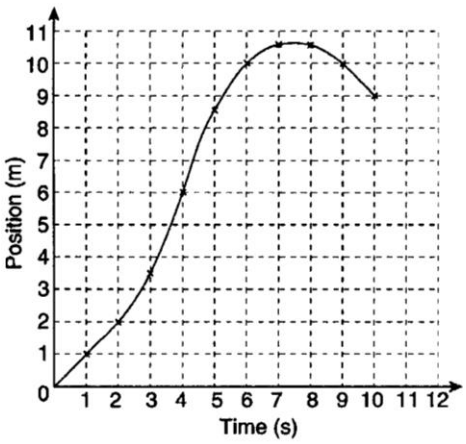
b Displacement is found from the area under the *v–t* graph from $t = 0$ s to $t = 7$:
$s = 10.5$ m north

c During first 10 s, the area under the graph = 10.5 – 1.5 = 9 m north.

11 **a** Displacement = area under graph between 2 s and 4 s:
Displacement = 4.0 m north
v_{av} = displacement/time = 4.0/2.0 = 2.0 m s^{-1} north

b Displacement = area under graph between 2s and 7 s = 8.5 m north
v_{av} = displacement/time = 8.5/5.0 = 1.7 m s^{-1} north

12



13 **a** The curve starts to flatten out at $t = 80$ s.

b The acceleration of the train at $t = 10$ s is determined from the gradient of the curve at that point, i.e. 1.3 m s^{-2}.

c The acceleration of the train at $t = 40$ s is determined from the gradient of the curve at that point, i.e. 0.5 m s^{-2}.

d Displacement of the train after 120 s is found by calculating the area under the *v–t* graph from $t = 0$ to $t = 120$ s:
Displacement = 4900 m = 4.9 km

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14 a The initial acceleration = rise/run = (8 – 0)/(4 – 0) = +2 m s^{-2}.

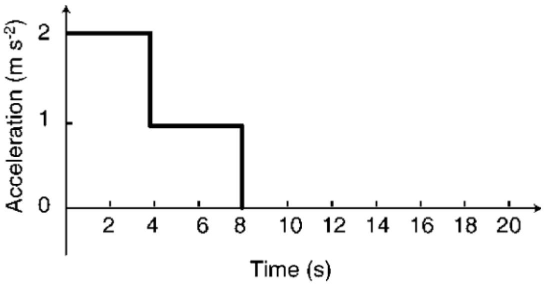
b The bus starts gaining ground on the bicycle when the respective velocities are equal. This occurs at $t = 4$ s.

c The bus will overtake the bicycle when the areas under the respective *v*–*t* graphs are equal. Using a counting squares approach, this occurs at $t = 10$ s.

d When $t = 10$ s the area under each *v*–*t* graph is 80 m.

e Average velocity = displacement/time = 56/8 = +7 m s^{-1}

15 a



b The change in velocity of the bus over the first 8 s is determined by calculating the area under the acceleration–time graph from $t = 0$ to $t = 8$ s, i.e. +12 m s^{-1}.

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4.3 Equations of motion

1 **a** $v = u + at$
$a = (v\text{-}u)/t = (16 – 0)/8.0$
$= 2.0\ \text{m s}^{-2}$

b Average velocity $= (u + v)/2 = (16 + 0)/2$
$= 8.0\ \text{m s}^{-1}$

c $x = ((u + v)/2)t = (16/2) \times 8.0 = 64\ \text{m}$

2 **a** $x = ut + ½at^2$
$400 = 0 + ½a(16)^2$
$a = 800/16^2 = 3.1\ \text{m s}^{-2}$

b $v = u + at$
$= 0 + (3.1 \times 16)$
$= 50\ \text{m s}^{-1}$

c $50\ \text{m s}^{-1} = \dfrac{50\times10^{-3}}{1/3600} = 180\ \text{km h}^{-1}$

3 **a** $v = u + at$
$160 = 0 + 4.0a$
$a = 40\ \text{m s}^{-2}$

b During first 4.0 s: $x = \left(\dfrac{u+v}{2}\right)t$
$(160/2) \times 4.0 = 320\ \text{m}$
From $t = 4$ s to $t = 9$ s: $x = v \times t = 160 \times 5.0 = 800\ \text{m}$
Total distance $= 320 + 800 = 1120\ \text{m} = 1.1 \times 10^3\ \text{m}$

c $160 \times 3.6 = 580\ \text{km h}^{-1}$

d v_{av} (0 s to 4 s) = distance travelled/time taken
$= 320/4.0 = 80\ \text{m s}^{-1}$

e v_{av} (0 s to 9 s) = distance travelled/time taken
$= 1120/9.0 = 124\ \text{m s}^{-1}$

4 **a** $v^2 = u^2 + 2ax$
$0^2 = 28.2^2 + 2 \times a \times 4.00$
$–795 = 8.00a$
$a = –99.4\ \text{m s}^{-2}$

b $x = \left(\dfrac{u+v}{2}\right)t$
$4.00 = \left(\dfrac{28.2+0}{2}\right)t$
$t = \dfrac{4.00}{14.1} = 0.284\ \text{s}$

c $v^2 = u^2 + 2ax$
$= 28.2^2 – 2 \times 99.4 \times 2$
$= 398$
$v = 19.9\ \text{m s}^{-1}$

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5 Cars have greatest accelerations when they are travelling slowly (i.e. when they are in a low gear). When they are travelling fast, they may have a high speed, but this speed does not increase rapidly when the throttle is pushed.

6 D is the correct answer because the stone is still moving with a downward velocity but is beginning to decelerate, which is acceleration in the opposite direction.

7 **a** $v = u + at$

$6.3 = 4.2 + 5.3 \times a$

$2.1 = 5.3a$

$a = \dfrac{2.1}{5.3} = 0.396 = +0.40\ \text{m s}^{-2}$

b $x = \left(\dfrac{u+v}{2}\right)t = \left(\dfrac{4.2+6.3}{2}\right) \times 5.3 = 27.825 = 28\ \text{m}$

c Average speed = $\dfrac{4.2+6.3}{2} = 5.25 = 5.3\ \text{m s}^{-1}$

8 **a** Speed = 75/3.6 = 21 m s^{-1}

b Travelling with constant speed so $x = v \times t = 21 \times 0.25 = 5.2$ m

c $v^2 = u^2 + 2ax$

$0 = (21)^2 - (2 \times 6.0)x$

$x = 37$ m

d Total distance = 37 + 5.2 = 42 m

9 **a** $v^2 = u^2 + 2ax$

$= 0 + 2(2.0 \times 4.0)$

$v = 4.0\ \text{m s}^{-1}$

b $v^2 = u^2 + 2ax$

$= 0 + 2(2.0 \times 8.0) = 5.7\ \text{m s}^{-1}$

c $v = u + at$

$4.0 = 0 + 2.0\text{t}$

$t = 2.0$ s

d $v = u + at$

$5.7 = 0 + 2.0t$

$t = 2.85$ s

The time taken to travel the final 4.0 m is 2.85 s – 2.0 s = 0.85 s.

10 **a** $v = u + at$

$12 = 0 + 1.5t$

$t = 8.0$ s

b The bus will catch up with the cyclist when they have each travelled the same distance from the point at which the cyclist first passes the bus.

Cyclist: constant velocity, so $x = 12 \times t$

Bus: uniform acceleration $u = 0$, $a = 1.5\ \text{m s}^{-2}$, $x = ?$, $t = ?$

$x = ut + ½at^2 = 0.75t^2$

When the bus catches up with the cyclist, their displacements are equal, so:

$12t = 0.75t^2$

$t = 16$ s

c $x = 12 \times 16 = 192$ m

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4.4 Vertical motion under gravity

1 **a** Aristotle would expect that the brick would fall much faster than the paperclip.

b Galileo would predict that they would fall together.

c They fall together.

2 The bubbles rise because they are an air element and are returning to their natural place in the atmosphere.

3 B is correct, as all bodies falling freely near the Earth's surface move with the same constant acceleration.

4 A and D are correct. A is correct because a vertically projected object is momentarily stationary at the top of its path; D is correct because the acceleration is due to gravity.

5 **a** **i** $v = u + at$
$= 0 + 9.8 \times 1.0$
$= 9.8 \text{ m s}^{-1}$

ii $v = u + at$
$= 0 + 9.8 \times 2.0$
$= 20 \text{ m s}^{-1}$

iii $v = u + at$
$= 0 + 9.8 \times 3.0$
$= 29 \text{ m s}^{-1}$

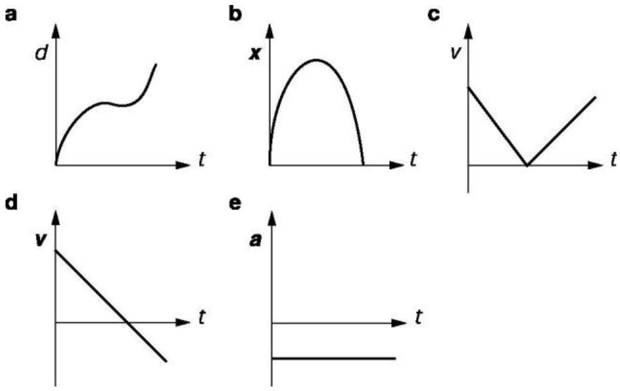
b $v^2 = u^2 + 2ax = 0 + 2 \times 9.8 \times 10 = 196$
$v = 14 \text{ m s}^{-1}$

c $v^2 = u^2 + 2ax = 0 + 2 \times 9.8 \times 20 = 392$
$v = 19.8 = 20 \text{ m s}^{-1}$

d $v^2 = u^2 + 2ax = 0 + 2 \times 9.8 \times 30 = 588$
$v = 24.2 = 24 \text{ m s}^{-1}$

e $v_{av} = (u + v)/2 = (0+24)/2 = 12 \text{ m s}^{-1}$

6



7 **a** $v = u + at = 0 + 9.8 \times 0.40 = 3.9 \text{ m s}^{-1}$

b $x = ut + ½at^2 = 0 + 0.5 \times 9.8 \times 0.402 = 0.78 \text{ m}$

c $x = ut + ½at^2 = 0 + 0.5 \times 9.8 \times 0.202 = 0.20 \text{ m}$

d $x = 0.78 – 0.20 = 0.58 \text{ m}$

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8 **a** It will reach maximum height after 2.0 s

b $x = vt - ½at^2 = 0 - 0.5 \times -9.8 \times 2.0^2 = 20$ m

c $v = u + at$
$u = 0 - (-9.8 \times 2.0) = 20$ m s^{-1}

d 20 m s^{-1}

e **i** 9.8 m s^{-2} down

ii 9.8 m s^{-2} down

iii 9.8 m s^{-2} down

9 **a** $x = vt - ½at^2$
$15.0 = 0 - 0.5 \times 9.8 \times t^2$
$t = 1.7$ s

b From maximum height of 15.0 m, the ball will fall by 11.0 m. Find how long it takes to travel this 15.0 m.

$x = ut + ½at^2$

$-11.0 = 0 + 0.5 \times -9.8 \times t^2$

$t = 1.5$ s

Total time from bounce = 1.7 + 1.5 = 3.2 s

10 The balloon travels at a constant 8.0 m s^{-1} for 80 m, so takes 10 s to reach the ground. The coin has an initial velocity of 8.0 m s^{-1}down and accelerates due to gravity. To calculate the time it takes to reach the ground, taking down as positive, use:
$x = ut + ½at^2$
$80 = 8.0 \times t + 0.5 \times 9.8 \times t^2$
Taking the positive answer to this quadratic gives $t = 3.3$ s, so the balloon reaches the ground 10 – 3.3 = 6.7 s after the coin.

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Chapter review

1 D, because the acceleration of the ball during its flight is always 9.8 m s^{-2} down.

2 $v = u + at$
$0 = u - (9.8 \times 1.5)$
$u = 15$ m s^{-1} up

3 $v^2 = u^2 + 2ax$
$0 = 14.7^2 - 2(9.8)x$
$x = 11$ m

4 Aristotle's theories did not explain why light objects fell as fast as heavy ones or why a solid (earth element) such as wood floated on water.

5 **a** The cyclist is travelling north during the interval where the graph has a positive gradient, i.e. between $t = 10$ s and $t = 25$ s.

b The cyclist is travelling south during the interval where the graph has a negative gradient, i.e. between $t = 30$ s and $t = 45$ s.

c The cyclist is stationary when the gradient of the graph is 0, i.e. between $t = 0$ s and $t = 10$ s, $t = 25$ s and $t = 30$ s, $t = 45$ s and $t = 60$ s.

6 The motorcyclist will pass back through the intersection when his/her displacement is zero again. This occurs at $t = 42.5$ s.

7 **a** Find gradient between 10 and 25 s:

v = rise/run
= (500 – 200)/15
= 20 m s^{-1} north

b Find gradient between 30 and 45 s:

v = rise/run
= (100 – 500)/10
= –40 m s^{-1}
= 40 m s^{-1} south

8 **a** Average velocity = displacement/time = –300/60 = –5.0 m s^{-1}
Magnitude = 5.0 m s^{-1}

b Average speed = total distance travelled/time taken
= (300+600)/60 = 900/60 = 15 m s^{-1}

9 $v^2 = u^2 + 2ax$
$0 = 10^2 + 2a \times 10$
$a = -5$ m s^{-2}
C is the correct answer.

10 $v = u + at$
$0 = 10 - 5t$
$t = 2.0$ s

11 Average speed = distance/time = $\dfrac{15\text{ km} + 5.0\text{ km} + 5.0\text{ km} + 5.0\text{ km}}{2.0\text{ h}}$ = 15 km h^{-1}

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12 **a** Average velocity = displacement/time
= 20/2.0 = 10 km h^{-1} north

b 10 km h^{-1} north = 10 km h^{-1} north/3.6 = 2.8 m s^{-1} north

13 **a** $x = ut + ½at^2$
$2.0 = 0 + ½a(1.0)^2$
a = 4.0 m s^{-2}

b $v = u + at = 0 + 4.0 \times 1.0 = 4.0$ m s^{-1}

c After 2.0 s the total distance travelled:
$x = ut + ½at^2 = 0 + 0.5 \times 4.0 \times 2.0^2 = 8.0$ m
Distance travelled during the second second = 8.0 m – 2.0 m = 6.0 m

14 **a** Average speed = distance/time
$= \dfrac{2\text{ cm} + 1.8\text{ cm} + 1.6\text{ cm} + 1.4\text{ cm} + 1.2\text{ cm}}{5 \times 0.02\text{ s}}$
= 8.0/0.10 = 80 cm s^{-1} = 0.80 m s^{-1}

b Average speed = distance/time
= 4.0/0.08 = 0.5 m s^{-1}

c average speed = distance/time
= 12/0.18 = 0.67 m s^{-1}

15 **a** $v = \dfrac{1.6\text{ cm} + 1.4\text{ cm}}{0.04\text{ s}}$ = 75 cm s^{-1} = 0.75 m s^{-1}

b x (of section A) = 0.08 m
v (at end of section A) = average peed of section B = 0.4 m s^{-1}
t (of section A) = 5/50 = 0.1 s
Use $x = vt – ½at^2$:
$0.08 = (0.4 \times 0.1) – ½a(0.1)^2$
$0.08 = 0.04 – 0.005a$
$a = 0.04 \div –0.005$
a = 8 m s^{-2}

16 **a** $x = ut + ½at^2$
$60.0 = 0 + 0.5 \times 9.8t^2$
t = 3.5 s

b $x = ut + ½at^2$
$70 = 10t + 0.5 \times 9.8t^2$
t = 2.9 s

17 The mass will overtake the shot at time t, where:
$10 + 4.9t^2 = 10t + 4.9t^2$
t = 1.0 s

18 The average velocity is estimated from (total displacement during the time interval)/11 s:
v_{av} = ~10 m s^{-1} north.

19 A. The velocity of the cyclist is always positive, indicating that he is always travelling north. B, C and D would only be correct if the graph was a displacement–time graph, instead of a v–t graph.

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20 **a** a = rise/run = 0

b a = rise/run
= (14 – 10)/2 = 2.0 m s^{-2} north

c a = rise/run
= (0 – 14)/2 = –7.0 m s^{-2} = 7.0 m s^{-2} south

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