Relations and functions Trial Test

Section A Multiple Choice

1 If A = {4, 8, 12, 16, 20} and B = {3, 6, 9, 12, 15, 18}, then $A \cup B$ is: B

A {3, 4, 6, 8, 9, 12, 12, 15, 16, 18, 20}
B {3, 4, 6, 8, 9, 12, 15, 16, 18, 20}
C {12}
D {4, 8, 16, 20}
E {3, 6, 9, 15, 18}

2 The appropriate number fields for π and $\sqrt{121}$ respectively are: A

A R and N
B Q and N
C R and J
D R and Q
E Q and J

3 Consider the following two graphs: B

Graph A $4x - 3y + 2 = 0$, where $x \in J$,

Graph B $y = \dfrac{4x^2}{3}$, where $-5 < x < 5$.

Which of the following is true?

A Both graphs are continuous.
B Graph A is discrete and Graph B is continuous.
C Both graphs are discrete.
D Graph A is continuous and Graph B is discrete.
E More information is needed before deciding whether a graph is discrete or continuous.

4 The domain and range of $y = \sqrt{3x+4}$, where $x < 4$, is: B

A $[-\infty,4)$ and $[0,4)$
B $[-\frac{4}{3},4)$ and $[0,4)$
C $(-\frac{4}{3},4)$ and $(0,4)$
D $[-\frac{4}{3},4]$ and $[0,4]$
E $[-\infty,4]$ and $[0,4]$

5 $R^+ \cup \{-2 \le x < 0\}$ in interval notation is: C

A $[-2, \infty)$
B $(-2, \infty)$
C $[-2, 0) \cup (0, \infty)$
D $(-2, \infty) / \{0\}$
E $(-2, \infty) \cup (0, \infty)$

6 Which one of the following sets of coordinates is a one-to-many relation? B

A {(–3, 2), (4, –2), (–3, –2), (4, 2)}
B {(1, 1), (1, 3), (1, 5)}
C {(1, 1), (2, 3), (3, 3)}
D {(2, –3), (3, –3), (4, –3)}
E {(1, 1), (2, 2), (3, 3)}

7 Which of the following relations are functions? A

I $y = 4x^2 + 1$
II $y = 2x - 3$
III $x^2 + y^2 = 20$
IIII $3x + 1 = y^2$

A I and II ?
B I, II and III ?
C I, II and IIII ?
D I and IIII ?
E II and IIII ?

8 If $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$, then $f(g(x))$ is: A

A $4x^2 + 18x + 17$
B $g(f(x))$
C $x^2 + 5x + 2$
D $2x^3 + 9x^2 + 7x - 3$
E $4x^2 + 6x + 17$

9 If $f(x) = 3x$ and $g(x) = x + 2$, then $g(f(-1))$ is: E

A -2

B -3

C 3

D -4

E -1

10 The maximal domain and range for $g(x) = \dfrac{4}{3+2x}$ are respectively: C

A $R \setminus \left\{\frac{3}{2}\right\}$ and $R \setminus \{0\}$

B $R \setminus \left\{-\frac{2}{3}\right\}$ and $R \setminus \{4\}$

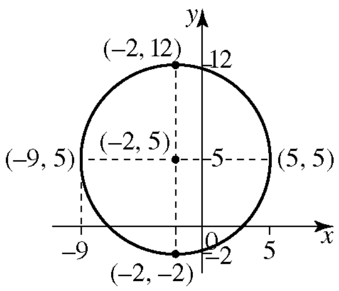
C $R \setminus \left\{-\frac{3}{2}\right\}$ and $R \setminus \{0\}$

D $R \setminus \left\{-\frac{2}{3}\right\}$ and $R \setminus \{0\}$

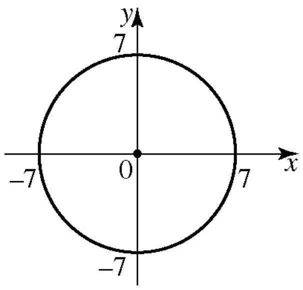
E $R \setminus \left\{\frac{2}{3}\right\}$ and $R \setminus \{4\}$

11 Which of the following best represents the circle with equation $(x+2)^2 + (y+5)^2 = 49$? E

A



B



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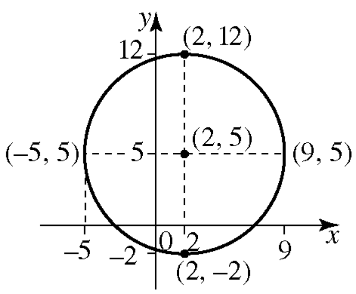

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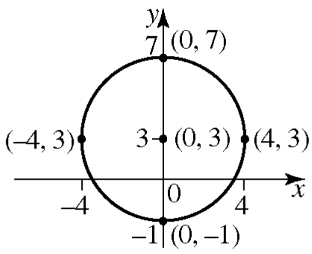
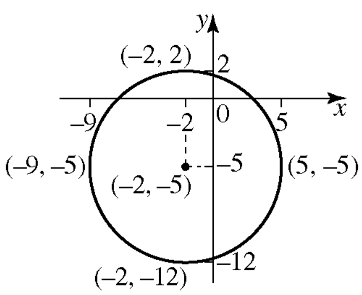


E



12 A rectangular pig sty is to be made using 20 metres of fencing. If x is the length of one side, then the domain of x is:

D

A (0, 20)

B [0, 20]

C [0, 10]

D (0, 10)

E (0, 5]

Relations and functions

Name: ______________________________

Section B Short/Extended answer

1 If $A = \{2, 4, 6, 8, 10\}$ **4**

$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and

$C = \{4, 8\}$, find:

(a) $(A \cap B) \cup C$

(b) $(A \cup B) \cap C$

(c) $A \setminus B$

(d) $B \setminus A$

(a) $(A \cap B) \cup C$

$= \{2, 4, 6, 8, 10\} \cup \{4, 8\}$

$= \{2, 4, 6, 8, 10\}$

(b) $(A \cup B) \cap C$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cap \{4, 8\}$

$= \{4, 8\}$

(c) $A \setminus B = \varnothing$

(d) $B \setminus A = \{1, 3, 5, 7, 9\}$

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2 Place each of the following into the appropriate number field. **5**

(a) $\sqrt{25}$

(b) $\dfrac{22}{7}$

(c) π^2

(d) -6.33333

(e) $\sqrt{90}$

(a) $\sqrt{25} = 5 \in N$ a natural number

(b) $\dfrac{22}{7} \in Q$ a rational number

(c) $\pi^2 \in R$ an irrational number

(d) $-6.33333 \in Q$ a rational number

(e) $\sqrt{90} \in R$ an irrational number

3 What are the domain and range of the following graphs? **8**

(a) $y = x^2 - 3$ where $x \in R$

(b) $y = \frac{1}{2}x$ where $x \in J$

(c) $y = \sqrt{3 - x}$, where $x \in \{2, 3, 4, 5, 6\}$

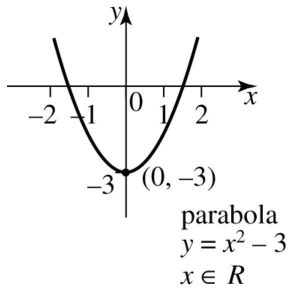
(d) $y = \frac{1}{x} + 3$, where $-3 < x < 3$

Sketch each of the above graphs.

(a) $y = x^2 - 3$ where $x \in R$

Domain $(-\infty, \infty)$

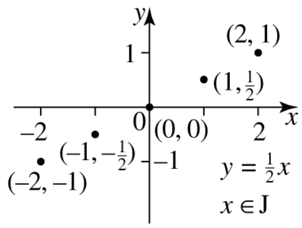
Range $[-3, \infty)$



(b) $y = \frac{1}{2}x$ where $x \in J$

Domain $x \in J$ $(-\infty, \infty)$

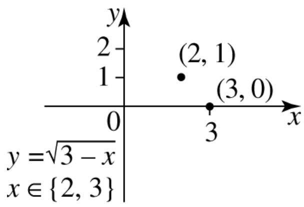
Range $y \in Q$ $(-\infty, \infty)$



(c) $y = \sqrt{3 - x}$, where $x \in \{2, 3, 4, 5, 6\}$

Domain $x \in \{2, 3\}$

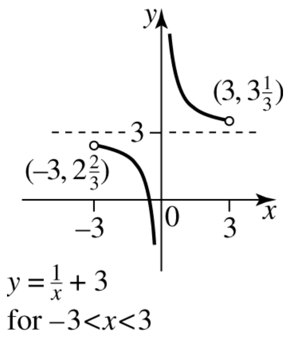
Range $y \in \{1, 0\}$



(d) $y = \frac{1}{x} + 3$, where $-3 < x < 3$

Domain $(-3,0) \cup (0,3)$

Range $\left(-\infty, 2\frac{2}{3}\right) \cup \left(3\frac{1}{3}, \infty\right)$



4 Rewrite these in interval notation. 3

(a) $R^+\backslash\{2\}$

(b) $R^+\backslash\{2 < x \le 6\}$

(c) $R^+ \cup \{-3 < x < -1\}$

(a) $R^+\backslash\{2\}$ $= (0, 2) \cup (2, \infty)$

(b) $R^+\backslash\{2 < x \le 6\}$ $= (0, 2] \cup (6, \infty)$

(c) $R^+ \cup \{-3 < x < -1\} = (-3, -1) \cup (0, \infty)$

5 Consider the following equations and determine the type of relation for each. 8

(a) $y = x^3$

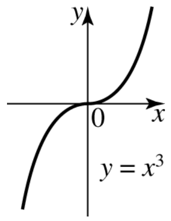
(b) $y = 3x - x^2$

(c) $x^2 + y^2 = 9$

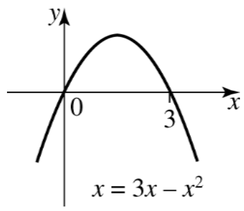
(d) $x = y^2 - 2y + 1$

Sketch each of the above graphs.

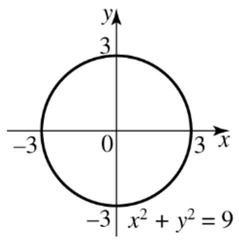
(a) $y = x^3$ one-to-one



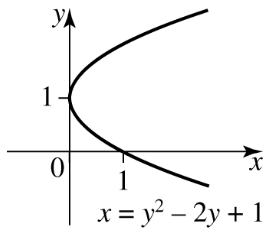
(b) $y = 3x - x^2$ many-to-one



(c) $x^2 + y^2 = 9$ many-to-many



(d) $x = y^2 - 2y + 1$ one-to-many



6 If $f(x) = 1 - x^2$ and $g(x) = x + 2$, find: 4

(a) $f(-1)$

(b) $g(3)$

(c) $f(g(x))$

(d) $g(f(x))$

(a) $f(-1) = 1 - (-1)^2 = 0$

(b) $g(3) = 3 + 2 = 5$

(c) $f(g(x)) = 1 - (x + 2)^2$
$= 1 - (x^2 + 4x + 4)$
$= 1 - x^2 - 4x - 4$
$= -x^2 - 4x - 3$

(d) $g(f(x)) = 1 - x^2 + 2^2$
$= 3 - x^2$

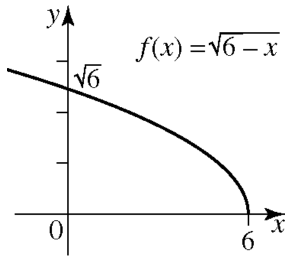
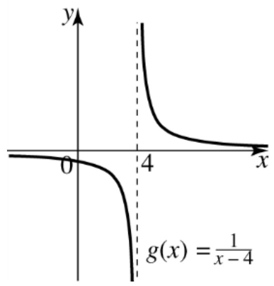
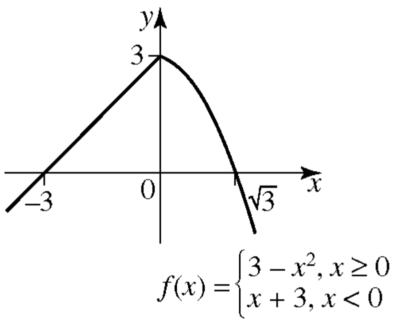
7 If $f(x) = 1 - 4x^2$ and $g(x) = 4x - 1$, find: 5

(a) $f(g(-3))$

(b) $g(f(-3))$

(a) $f(g(x)) = 1 - 4(4x - 1)^2$
$= 1 - 4(16x^2 - 8x + 1)$
$= 1 - 64x^2 + 32x - 4$
$f(g(-3)) = -64(-3)^2 + 32(-3) - 3$
$= -576 - 96 - 3$
$= -675$

(b) $g(f(x)) = 4(1 - 4x^2) - 1$
$= 4 - 16x^2 - 1$
$gf(-3) = 3 - 16(-3)^2$
$= -141$

8	What are the maximal domain and range for each of the following? (a) $f(x) = \sqrt{6-x}$ (b) $g(x) = \dfrac{1}{x-4}$ Sketch each of the above graphs.	(a) $f(x) = \sqrt{6-x}$ Domain $(-\infty, 6]$ Range $[0, \infty)$  (b) $g(x) = \dfrac{1}{x-4}$ Domain $R\backslash\{4\}$ Range $R\backslash\{0\}$  	6
9	If $f(x) = \begin{cases} 3 - x^2 & , \quad x \geq 0 \\ x + 3 & , \quad x < 0 \end{cases}$ find the range of $f(x)$ and the value for $f(-2)$. Sketch this graph.	Range $(-\infty, 3]$ and $f(-2) = -2 + 3 = 1$  	4

10 For the function with the rule $f(x)=2x^3-3x^2+x-8$, find: 9

(a) $f(-1)$

(b) $f(2)$

(c) $f(a)$

(d) $f(x+h)$

(e) Simplify $\dfrac{f(x+h)-f(x)}{h}$.

(f) Let $h = 0$ to simplify the final expression in part (e). The final answer is called the derivative of $f(x) = f'(x)$.

$f(x)=2x^3-3x^2+x-8$

(a) $f(-1) = 2(-1)^3 - 3(-1)^2 + (-1) - 8$
$= -2 - 3 - 1 - 8$
$= -14$

(b) $f(2) = 2(2)^3 - 3(2)^2 + (2) - 8$
$= 16 - 12 + 2 - 8$
$= -2$

(c) $f(a)=2a^3-3a^2+a-8$

(d) $f(x+h)=2(x+h)^3-3(x+h)^2+(x+h)-8$
$= 2x^3 + 6x^2h + 6xh^2 + 2h^3$
$-3x^2 - 6xh - 3h^2 + x + h - 8$

(e) $f(x + h) - f(x)$

$= 2x^3+6x^2h+6xh^2+2h^3-3x^2-6xh$

$-3h^2+x+h-8-2x^3+3x^2-x+8$

$=6x^2h+6xh^2+2h^3-6xh-3h^2+h$

$\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{6x^2h+6xh^2+2h^3-6xh-3h^2+h}{h}$

$=\dfrac{h(6x^2+6xh+2h^2-6x-3h+1)}{h}$

$=6x^2+6xh+2h^2-6x-3h+1$

(f) When $h = 0$, $f'(x)=6x^2-6x+1$

11 If $f(x)=x^2-3x$ and $g(x)=\sqrt{3x-2}$, find the rule for: 6

(a) $f(g(x))$

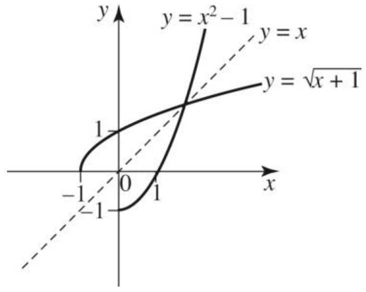
(b) $g(f(x))$

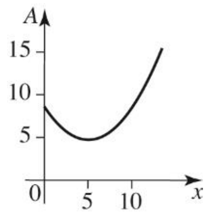
(c) $f(f(x))$

(a) $f(g(x)) = \left(\sqrt{3x-2}\right)^2-3\sqrt{3x-2}$

(b) $g(f(x))=\sqrt{3(x^2-3x)-2}$
$=\sqrt{3x^2-9x-2}$

(c) $f(f(x))=(x^2-3x)^2-3(x^2-3x)$
$=x^4-6x^3+9x^2-3x^2+9x$
$=x^4-6x^3+6x^2+9x$

12	If $g(f(x)) = (x^2 - 6)^3$, write down possibilities for $f(x)$, $g(x)$ and $f(g(x))$.	$g(x) = x^3$ $f(x) = x^2 - 6$ $f(g(x)) = (x^3)^2 - 6 = x^6 - 6$	4
13	Sketch the graph of $y = x^2 - 1$, $x \geq 0$. Hence sketch its inverse.		

14	A piece of wire 12 cm long is to be cut into two pieces. One piece will make a circular ring and the other a square pendant. Let x be the length of the piece to be made into a ring. Derive an expression in terms of x to denote the total area of the two shapes. What is the domain of x and the range of the total area?	Ring length = x Circumference of ring = $2\pi r$, so we have $x = 2\pi r$ $r = \frac{x}{2\pi}$ Area of ring = $\pi r^2 = \pi \frac{x^2}{4\pi^2} = \frac{x^2}{4\pi}$ Length of side of square = $\frac{12-x}{4}$ Area of square $= \frac{12-x}{4} \times \frac{12-x}{4}$ $= \frac{144-24x+x^2}{16}$ Total area = $\frac{x^2}{4\pi} + \frac{144-24x+x^2}{16}$ Domain of x is (0, 12) cm. Range of area is found by sketching graph (plot points or use graphing software). Area (vertical axis) versus x (horizontal axis):  When x = 0 cm, area = 9 cm^2 When x = 12 cm, area = $\frac{36}{\pi} \approx 11.5$ cm^2 When $x \approx 5.3$ cm, minimum area ≈ 5.0 cm^2 (Use a TRACE facility if possible.) Range $\approx$ (5, 11.5) cm^2	6