

WS # 1

Quadratic functions Solutions

Name: _____

- 1 Factorise and solve for x :

$$x^2 + 2x - 6 = 0$$

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$$x^2 + 2x + 1 - 1 - 6 = 0$$

$$(x+1)^2 - 7 = 0$$

$$(x+1-\sqrt{7})(x+1+\sqrt{7}) = 0$$

$$x = -1 + \sqrt{7} \text{ or } x = -1 - \sqrt{7}$$

- 2 Factorise and solve for x :

$$3x - x^2 = 1$$

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$$0 = x^2 - 3x + 1$$

$$x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 1 = 0$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{5}{4} = 0$$

$$\left(x - \frac{3}{2} - \frac{\sqrt{5}}{2}\right)\left(x - \frac{3}{2} + \frac{\sqrt{5}}{2}\right) = 0$$

$$x = \frac{3}{2} + \frac{\sqrt{5}}{2} \text{ or } x = \frac{3}{2} - \frac{\sqrt{5}}{2}$$

- 3 Use the quadratic formula to find exact values for x in the equation $3x^2 - x - 1 = 0$.

$$3x^2 - x - 1 = 0$$

$$a = 3, b = -1, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1+12}}{6}$$

$$x = \frac{1 + \sqrt{13}}{6} \text{ or } x = \frac{1 - \sqrt{13}}{6}$$

- 4 Use the quadratic formula to find exact values for x given $3 - 2x^2 = 7x$.

$$\begin{aligned}3 - 2x^2 &= 7x \\ -2x^2 - 7x + 3 &= 0 \\ a &= -2, b = -7, c = 3 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{7 \pm \sqrt{49 + 24}}{-4} \\ x &= \frac{7 + \sqrt{73}}{-4} \quad \text{or} \quad x = \frac{7 - \sqrt{73}}{-4} \\ x &= \frac{-7 - \sqrt{73}}{4} \quad \text{or} \quad x = \frac{-7 + \sqrt{73}}{4}\end{aligned}$$

- 5 Find the value for the discriminant in $-5x^2 + x - 3 = 0$.

$$\begin{aligned}-5x^2 + x - 3 &= 0 \\ a &= -5, b = 1, c = -3 \\ \Delta &= b^2 - 4ac \\ &= 1 - 4 \times -5 \times -3 \\ &= 1 - 60 \\ &= -59\end{aligned}$$

- 6 Use the value for the discriminant to determine the number and type of solutions for $7x^2 - 3x - 5 = 0$.

$$\begin{aligned}7x^2 - 3x - 5 &= 0 \\ a &= 7, b = -3, c = -5 \\ \Delta &= b^2 - 4ac \\ &= 9 - 4 \times 7 \times -5 \\ &= 9 + 140 \\ &= 149\end{aligned}$$

Since $\Delta > 0$, the equation has two real solutions.

- 7 For what values of k does the following equation have just one solution?
 $(4k + 1)x^2 + 3kx + 1 = 0$

$$\begin{aligned}(4k + 1)x^2 + 3kx + 1 &= 0 \\ a &= 4k + 1, b = 3k, c = 1 \\ \Delta &= b^2 - 4ac \\ &= 9k^2 - 4 \times (4k + 1) \times 1 \\ &= 9k^2 - 16k - 4 \\ \text{For one solution, require } \Delta &= 0. \\ 0 &= (9k + 2)(k - 2) \\ k &= -\frac{2}{9} \quad \text{or} \quad k = 2 \quad \text{for one solution}\end{aligned}$$

- 8 Sketch the parabola with equation $y = -2(x-3)^2 + 4$, showing clearly the turning point and y-intercept.

$$y = -2(x-3)^2 + 4$$

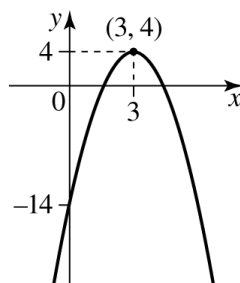
A negative coefficient of x^2 means a maximum turning point.

The coordinates of the turning point are (3, 4).

y-intercept: when $x = 0$,

$$y = -2(-3)^2 + 4$$

$$y = -14$$



- 9 Convert the following quadratic into turning point form to determine the coordinates of the turning point:

$$y = x^2 + 6x - 11$$

Sketch the parabola, showing all intercepts and the turning point.

$$y = x^2 + 6x - 11$$

$$y = x^2 + 6x + 9 - 9 - 11$$

$$y = (x+3)^2 - 20$$

The turning point is (-3, -20).

A positive coefficient of x^2 means a minimum turning point.

y-intercept: when $x = 0$, $y = -11$

x-intercept: when $y = 0$, $0 = x^2 + 6x - 11$

$$a = 1, b = 6, c = -11$$

$$x = \frac{-6 \pm \sqrt{36 + 44}}{2}$$

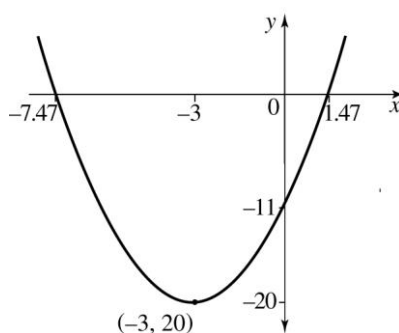
$$= \frac{-6 \pm \sqrt{80}}{2}$$

$$= \frac{-6 \pm 4\sqrt{5}}{2}$$

$$= -3 \pm 2\sqrt{5}$$

$$x = -3 + 2\sqrt{5} \text{ or } x = -3 - 2\sqrt{5}$$

$$x \approx 1.47 \text{ or } x \approx -7.47$$



- 10** The graphs with equations $y = x^2 + 5x - 9$ and $y = cx - 10$ intersect once only. Find the possible values of c .

$$y = x^2 + 5x - 9 \quad [1]$$

$$y = cx - 10 \quad [2]$$

$$x^2 + 5x - 9 = cx - 10$$

$$x^2 + 5x - cx + 1 = 0$$

$$x^2 + (5 - c)x + 1 = 0$$

$$\Delta = (5 - c)^2 - 4 \times 1$$

$$= 25 - 10c + c^2 - 4$$

$$= c^2 - 10c + 21$$

$$\Delta = 0, \text{ for one solution only.}$$

$$c^2 - 10c + 21 = 0$$

$$(c - 7)(c - 3) = 0$$

$$c = 7 \quad \text{or} \quad c = 3$$